PATH GENERATION OF A PLANAR SIX-BAR LINKAGE

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Abstract: *The planar path generator is often used as a sub-chain of a six bar linkage that is used to produce motions which cannot be produced by a simple four-bar linkage. In this paper the kinematic optimization of a six-bar mechanism has been explained using a numerical approach based on the AMESim Optimization tool. The optimization procedure can be done when the motion of the mechanism is given end dimensions of the elements are to be calculated.*

Keywords: six-bar linkage, simulation, path generation, optimization procedure, genetic algorithm

INTRODUCTION

Kinematics synthesis, in general, implies the development of methods of computation or graphical construction, that implicitly determine the proper dimensions for the synthesized mechanism. A design process is a sequence of decisions, each of which must be evaluated and altered as necessary. The design engineer brings to this process his experience from past successful designs. The ultimate goal of the design process is the discovery of the optimum solution for a given design situation.

The system we are going to model is a six-bar planar linkage (see figure below). Body 1 is connected to the ground at point O_1 through a pivot joint. It is also connected to body 2-6 with the pivot joint A. Body 2-6 is connected to body 4 with the pivot joint B and to body 3-7 with the pivot joint C. The bodies 4 and 5 are connected to the ground at pivot joints O_2 and O_3. The body 3-7 is connected to body 5 with the pivot joint E. The motion of coupler point D is defined when the position vector of this point are defined as function of time with respect to a fixed reference frame with the origin at O.

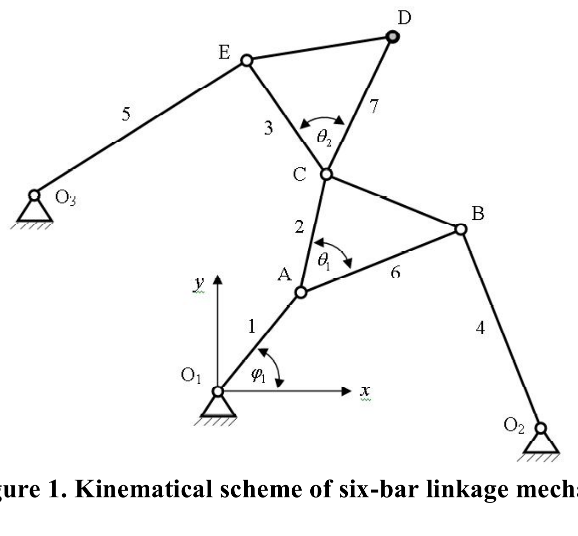

Figure 1. Kinematical scheme of six-bar linkage mechanism

EQUATIONS OF MOTION

The coordinates of joint A are expressed in terms of the coordinates of joint O_1 and the relative orientation of link 1. Its coordinates are determined using following equations

$$x_A = x_{O1} + l_1 \cos \varphi_1 \quad (1)$$

$$y_A = y_{O1} + l_1 \sin \varphi_1 \quad (2)$$

For the joint B on the links 6 and 4 can be writing

$$x_B = x_A + l_6 \cos\varphi_6 = x_{O2} + l_4 \cos\varphi_4 \quad (3)$$
$$y_B = y_A + l_6 \sin\varphi_6 = x_{O2} + l_4 \sin\varphi_4 \quad (4)$$

where φ_4 and φ_6 are the relative orientations of links 4 and 6 with respect to axis ox of the Cartesian reference frame *Oxy*.
By eliminating φ_4 combining equation (3) with (4) and summarizing we have

$$[(x_A - x_{O2}) + l_6 \cos\varphi_6]^2 + [(y_A - y_{O2}) + l_6 \sin\varphi_6]^2 = l_4^2 \quad (5)$$

or

$$A\cos\varphi_6 + B\sin\varphi_6 + C = 0 \quad (6)$$

Where

$$A = 2l_6(x_A - x_{O2}) \quad (7)$$
$$B = 2l_6(y_A - y_{O2}) \quad (8)$$
$$C = (x_A - x_{O2})^2 + (y_A - y_{O2})^2 + l_6^2 - l_4^2 \quad (9)$$

With notation

$$T = \tan\frac{\varphi_6}{2} \quad (10)$$

equation (6) can be writing as

$$(C - A)T^2 + 2BT + C + A = 0 \quad (11)$$

and is obtained

$$T = \frac{-B \pm \sqrt{B^2 - C^2 + A^2}}{C - A} \quad (12)$$

respective

$$\varphi_6 = 2\arctan T \quad (13)$$
$$\varphi_2 = \varphi_6 + \theta_1 \quad (14)$$

Similarly, for links 3-7, 5 and joints C, E, and couplet point D can write relations

$$x_C = x_A + l_2 \cos\varphi_2 \quad (15)$$
$$y_C = y_A + l_2 \sin\varphi_2 \quad (16)$$
$$x_E = x_C + l_3 \cos\varphi_3 = x_{O3} + l_5 \cos\varphi_5 \quad (17)$$
$$y_E = y_C + l_3 \sin\varphi_3 = y_{O3} + l_5 \sin\varphi_5 \quad (18)$$
$$A'\cos\varphi_3 + B'\sin\varphi_3 + C' = 0 \quad (19)$$
$$A' = 2l_3(x_C - x_{O3}) \quad (20)$$
$$B' = 2l_3(y_C - y_{O3}) \quad (21)$$
$$C' = (x_C - x_{O3})^2 + (y_C - y_{O3})^2 + l_3^2 - l_5^2 \quad (22)$$
$$T' = \tan\frac{\varphi_3}{2} \quad (23)$$
$$(C' - A')T'^2 + 2B'T' + C' + A' = 0 \quad (24)$$
$$T' = \frac{-B' \pm \sqrt{B'^2 - C'^2 + A'^2}}{C' - A'} \quad (25)$$

respective

$$\varphi_3 = 2\arctan T' \quad (26)$$
$$\varphi_7 = \varphi_3 - \theta_2 \quad (27)$$

For the coupler point D, can be writing

$$x_D = x_C + l_7 \cos\varphi_7 \quad (28)$$
$$y_D = y_C + l_7 \sin\varphi_7 \quad (29)$$

For the synthesis of this mechanism it starts from some initial values of dimensional array parameters l_1, l_2, l_3, l_4, l_5, l_6, l_7, φ_1, θ_1, θ_2, x_{O1}, y_{O1}, x_{O2}, y_{O2}, x_{O3}, y_{O3} and for a desired trajectory of point D, imposed by an optimization algorithm to determine those values of the above mentioned parameters, that the really trajectory of coupler point D is as close to the desired trajectory. The dates defining the desired trajectory are presented in table 1.

Table 1.

$\varphi_1[rad]$	**0.91**	**1.13**	**1.25**	**1.39**	**1.41**	**1.47**	**1.52**	**1.59**	**1.63**	**1.68**	**1.74**	**1.79**	**1.86**	**1.92**
$x_{D-dez}[m]$	0.07	0.09	0.11	0.13	0.15	0.17	0.19	0.21	0.23	0.25	0.27	0.29	0.31	0.33
$y_{D-dez}[m]$	0.02	0.07	0.11	0.14	0.166	0.186	0.2	0.22	0.23	0.24	0.25	0.26	0.27	0.275

SIMULATION AND OPTIMIZATION PROCEDURES

Based on kinematical scheme and calculus relations are building the AMESim models of simulation scheme (figure 2) and optimization scheme (figure 3) of linkage mechanism. The optimization procedure adopted is Genetic Algorithms.

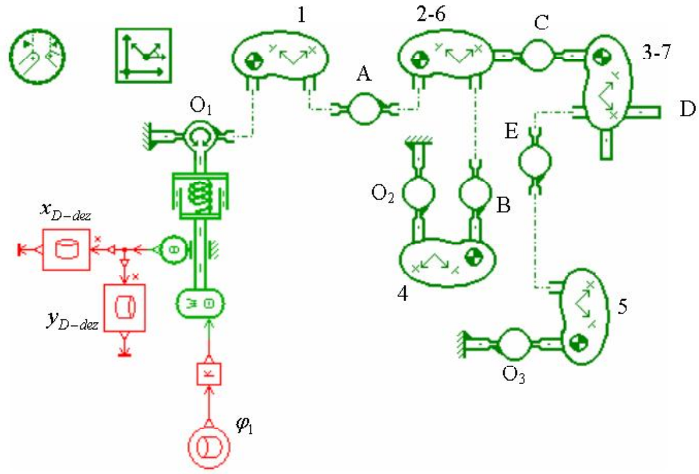

Figure 2. AMESim model of six-bar linkage mechanism

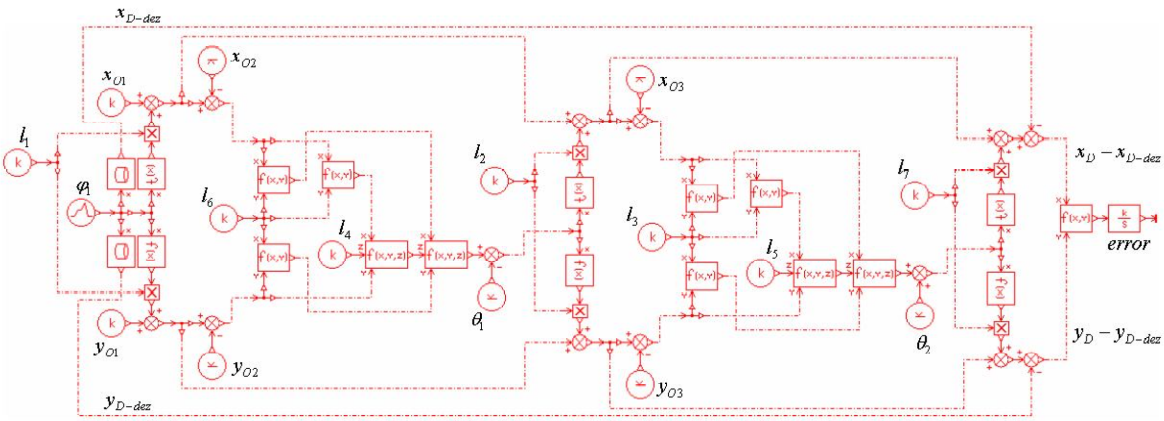

Figure 3. The model of linkage mechanism constructed using signal elements

Genetic Algorithms are adaptive heuristic search algorithm based on the evolutionary ideas of natural selection and genetics. As such they represent an intelligent exploitation of a random search used to solve optimization problems. Although randomised, Genetic Algorithms are by no means random, instead they exploit historical information to direct the search into the region of better performance within the search space. The basic techniques of the Genetic Algorithms are designed to simulate processes in natural systems necessary for evolution, specially those follow the principles first laid down by Charles Darwin of *survival of the fittest*. Since in nature, competition among individuals for scanty resources results in the fittest individuals dominating over the weaker ones.

Genetic Algorithms simulate the survival of the fittest among individuals over consecutive generation for solving a problem. Each generation consists of a population of character strings that are analogous to the chromosome. Each individual represents a point in a search space and a possible solution. The individuals in the population are then made to go through a process of evolution.

For optimization it is starts at the next initial configuration: $x_{O1}=0[m]; y_{O1}=0[m]$; $x_{O2}=-0.1[m]; y_{O2}=0.55[m]$; $x_{O3}=0.15[m]; y_{O3}=-0.4[m]$; $l_1=0.05[m]$; $l_2=0.1[m]$; $l_3=0.1[m]$; $l_4=0.5[m]$; $l_5=0.5[m]$; $l_6=0.16[m]$; $l_7=0.07$ [m]; $\theta_1=1.25 rad$; $\theta_2=0.78 rad$.

The objective function is defined as

$$Obj=\int_{0.91}^{1.92}\left[\left(x_D-x_{D-dez}\right)^2+\left(y_D-y_{D-dez}\right)^2\right]d\varphi_1 \tag{30}$$

The results obtained by Genetic Algorithms after 40 generations are indicated in the figure below

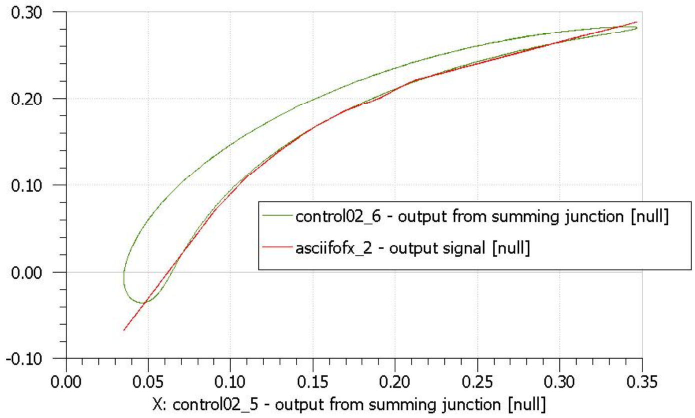

Figure 4. The optimal solutions found by the optimization algorithm: a) dimensional values; b) coupler point curves.

$x_{O1}=0.237857[m]$; $y_{O1}=0.002953[m]$; $x_{O2}=-0.062258[m]$; $y_{O2}=0.029556[m]$; $x_{O3}=0.227724[m]$; $y_{O3}=-0.058804[m]$; $l_1=0.041168[m]$; $l_2=0.233601[m]$; $l_3=0.397101[m]$; $l_4=0.343228[m]$; $l_5=0.368362[m]$; $l_6=0.450078[m]$; $l_7=0.434057[m]$; $\theta_1=-0.259452[rad]$; $\theta_2=-0.695286[rad]$.

CONCLUSIONS

As can be seen in the figure above, between required and obtained values by optimization procedure is a very good coincidence. The approximation error, that is the minimum value of the objective function, is 0.165795 . Accordance with the procedure described above can be solved with success many of the practical problems of synthesis of planar mechanisms. Can be taken into consideration a multitude of solutions in a short time and optimum solutions results have a high degree of accuracy.

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