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# THE STUDY OF THE SYSTEMS USED FOR THE POWER SOURCES' COUPLING

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***Abstract:*** *In this paper we perform a review of the principles and technical solutions used for the power sources' coupling. For a mechanical system that uses a planetary two degrees of mobility system the authors realize the kinematic and dynamic analysis to obtain the equations of motion. Finally, for an automobile equipped with such a planetary mechanism, we determine the equations of motion, equation that may be numerically integrated.*

**Keywords**: power sources, hybrid automobile, planetary mechanism, Lagrange's equations

## INTRODUCTION

The coupling of the power sources is mainly realized for the propulsion of the hybrid automobile, where the propulsion energy is provided by two sources based on different principles for the energy's generation.
In the compounding of such a system three main components are included: a source of irreversible energy, a source of reversible energy, and a coupling system that permits the transmission of the motion to the automotive wheels (Fig. 1).

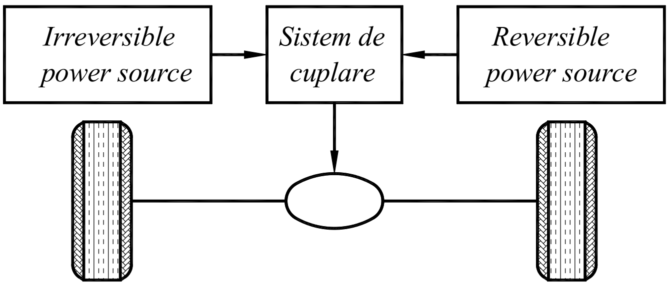


**Figure 1.** Hybrid propulsion system.

The source of irreversible energy is, usually, a system consisting in a reservoir of fuel and a thermal engine or a generator with fuel cell. Hence, the chemical energy is transformed in mechanical energy.
The source of reversible has also two components: a system for energy stocking and a source of recharging. The stocking system may be: an electro-chemical accumulator, a super- capacitor, an inertial flywheel or an oleo-pneumatic system. The recharging source permits the introduction of the energy in the system.
The coupling system assures the coupling of the two energetic sources and the transmission of the motion to the automotive wheel. This system may have mechanical or electric components.

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## MECHANICAL COUPLING SYSTEMS

Planetary mechanisms are used in almost every mechanical systems to couple the powers given by a thermal engine and two electric machines. This is the solution used by Toyota Company for its model Prius. The system is symbolized THS (Toyota Hybrid System) or, newer, HSD (Hybrid Synergy Drive). In Fig. 2 is presented the kinematic scheme of the mechanical coupling system.
The planetary mechanism denoted by $PSD$ (Power Split Device) realizes a division of the power stream. The advantage of such a solution is simplicity, reliability and robustness. The thermal engine $MT$ is linked to the port-satellite arm 1, the electric machine $MG_1$ to the solar wheel 3, and the electric machine $MG_2$ to the crown gear 4.

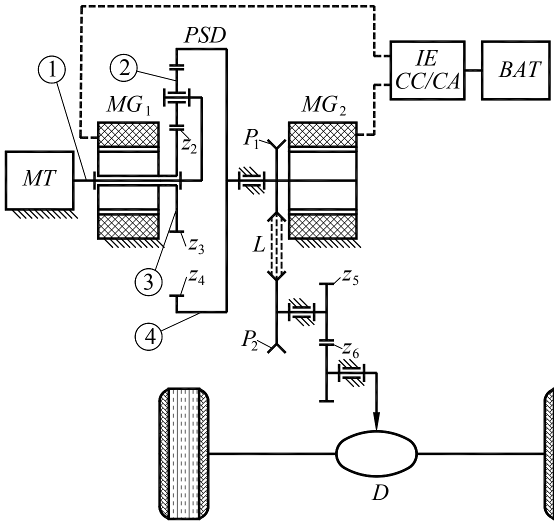


**Figure 2.** Mechanical coupling system. Toyota Prius solution.

The electric machines $MG_1$ and $MG_2$ are used either as motors, or generators, depending on the velocity of the automobile. They are coupled to the electric battery $BAT$ of high voltage (274 V) by an electronic inverter $IE$. The motion is transmitted using a chain gearing $L$, from the pinion $P_1$ fixed to rotation to the axle of the electric motor $MG_2$, to the pinion $P_2$, and, further on by the gear $z_5 - z_6$ to the motor wheels, using the differential gear $D$.
In Fig. 2, by dashed lines, were drawn the electric connections, and by continuous line, the mechanical ones.

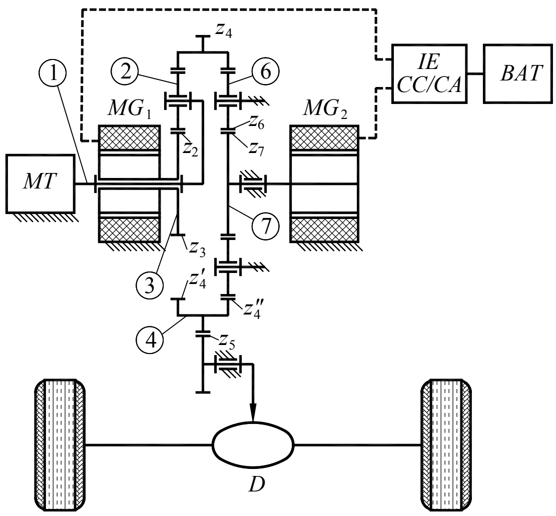


**Figure 3.** Mechanical coupling system. Lexus LS-600H solution.

A similar planetary mechanism is used by Lexus LS-600H automobile. As opposed to the mechanism of Toyota solution, this planetary mechanism is a double one, the second planetary mechanism (Fig. 3) consisting in the crown gear 4 ($z_4''$ teeth), planetary pinions 6 ($z_6$ teeth), and the solar wheel 7 ($z_7$ teeth) fixed to rotation to the electric machine $MG_2$. This mechanism having the satellites axes 6 fixed becomes a speed-reducer and, therefore, it amplifies the transmitted motor torque by the electric machine $MG_2$. The motion is transmitted by the gear $z_4 - z_5$ to the main transmission of the automobile.

## HYDRAULIC HYBRID SYSTEMS

The hydraulic hybrid systems take the mechanical energy from the thermal engine using a hydraulic machine which works as hydraulic motor in the inrush phase and as brake in braking phase. Such systems are mainly used for the medium and heavy automotive propulsion.

The best solution of such a system is that of the Australian Company Permo Drive Technologies, which, with the RDS model (Regenerative Drive Systems) equipped the military automotive FMTV (Family Medium Tactical Vehicles) with six motor wheels and a transmission with gears permitting a reduction of fuel of about 27% - 37%. In the same time, were reached the acceleration performance by 36% and braking ones by 60%.

The hydraulic hybrid systems may be series (Fig. 4, *a*) or parallel (Fig. 4, *b*) systems.

The series hydraulic systems (Fig. 4, *a*) consist in a hydraulic machine $HPM$, a low pressure accumulator $LPA$, a high pressure accumulator $HDA$ and a hydraulic acting system $HDA$. This assembling contains a hydraulic motor that converts the stocked hydraulic energy into mechanical energy. The necessary energy for the system is provided by the thermal engine $MT$.

The parallel hydraulic systems (Fig. 4, *b*) recover the kinetic energy in braking and convert it in hydraulic energy; they stock this energy to be used at the inrush or acceleration phase. The system consists in a hydraulic machine $HPM$, a reservoir of low pressure fluid $FR$, and an hydraulic accumulator of high pressure $HPA$ with nitrogen pillow (to stock the recovered hydraulic energy). the power stream closes, in both direction, through the front mechanical transmission $SM_F$, thermal engine $MT$, rear mechanical transmission $SM_S$, and the front and rear motor wheels.

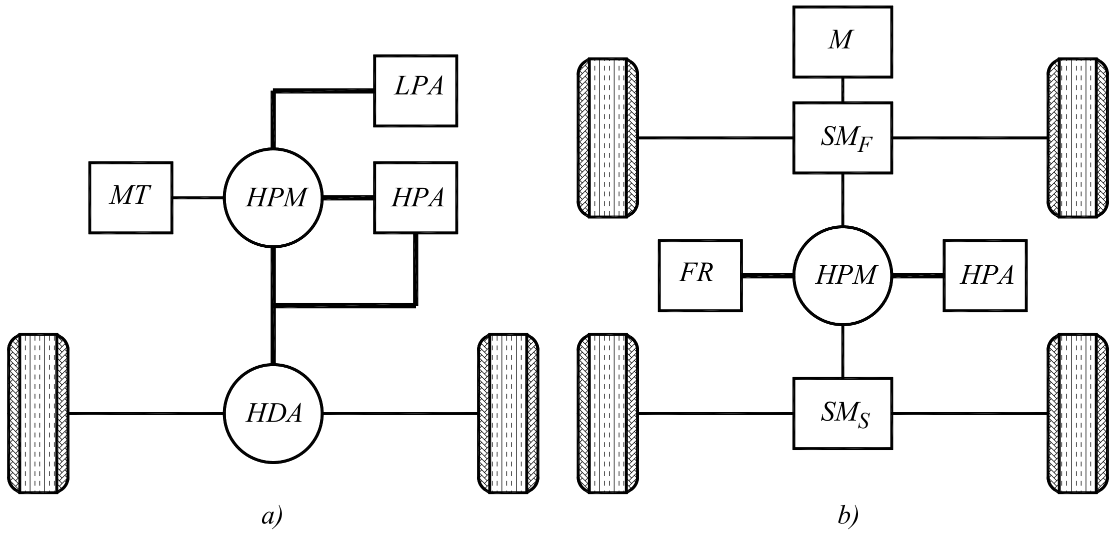


**Figure 4.** Hydraulic hybrid systems

## ELECTRIC COUPLING SYSTEMS

In this case the acting of the motor wheels is performed by the electric motors, the thermal engine having the role to provide energy in the system.

There are several solutions. In the case of the series hybrid in Fig. 5, *a*, the thermal engine $MT$ acts the electric generator $GE$, the energy thus obtained is stocked in the traction battery $BAT$. The stocking of the energy and then its use is controlled by the electronic inverter $IE$. The reversible electric machine $MG$ assures the propulsion (in motor regime) and the recovery of the energy at

braking (in generator regime). The mechanical energy of the thermal engine $MT$ completely passes to the electric generator $GE$ that converts it in electric and then electro-chemical energy in the battery $BAT$. This transformation reduces the efficiency to 70% - 80%.
In the case of the series hybrid without the stocking of energy at the braking in Fig. 5, *b*, the system is simpler. One uses one electric motor $ME$ and one planetary gear $RP$ for each motor wheel. The thermal engine acts the electric generator $GE$, and a command modulus $IC$ assures the electric energy to the motors at wheels. Such solutions are used at the electric Diesel engines (since more than 60 years) and some military 8x8 automotive.

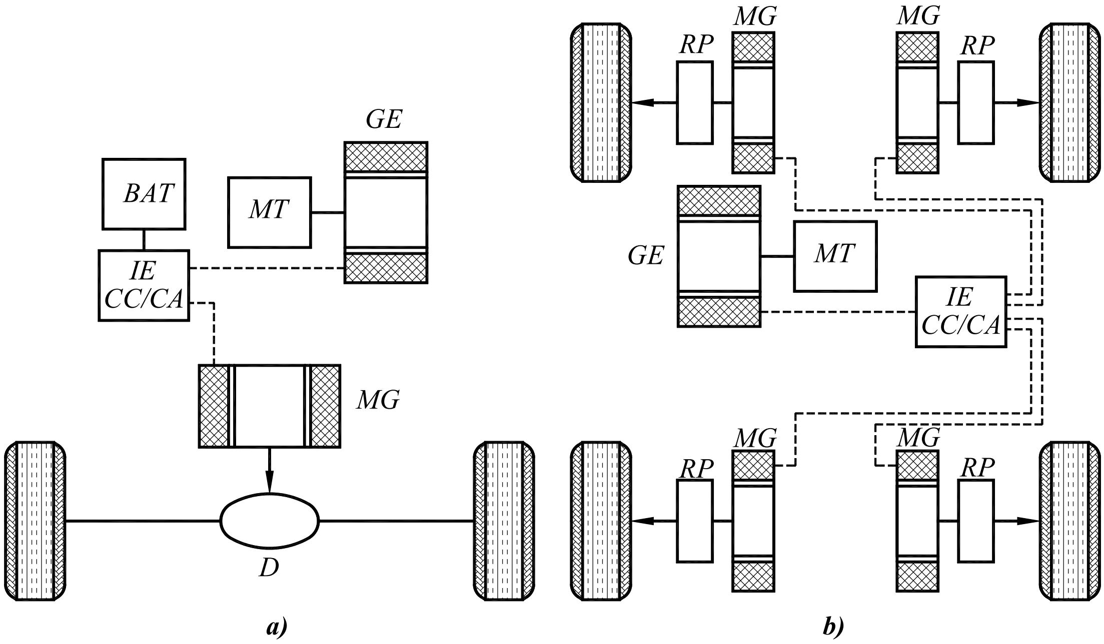


**Figure 5.** Series electric hybrid systems

## THE EQUATIONS OF MOTION OF THE MECHANICAL SYSTEMS USED IN THE POWER SOURCES' COUPLING

As we mentioned in paragraph 2, the mechanical systems use a planetary mechanism with two degrees of mobility to couple three power sources: a thermal source a two electric sources. We will analyze the solution captured in Fig. 2.
We denote by $z_2$, $z_3$ and $z_4$ the number of teeth of the gears 2, 3, and 4, respectively. To be out to obtain the kinematic relations, we will apply the Willis relation for the study of the relative motion with respect to the port-satellite 1. If we will denote by $\omega_2$, $\omega_3$ and $\omega_4$ the absolute angular velocities of the satellite pinion 2, planetary pinion 3, and gear crown 4, respectively, and by $\omega_1$ the absolute angular velocity of the port-satellite 1, we obtain for the mechanism in Fig. 2 the relations

$$\frac{\omega_3 - \omega_1}{\omega_2 - \omega_1} = -\frac{z_2}{z_3},\ \frac{\omega_2 - \omega_1}{\omega_4 - \omega_1} = \frac{z_4}{z_2}, \tag{1}$$

which lead to

$$\frac{\omega_3 - \omega_1}{\omega_4 - \omega_1} = -\frac{z_4}{z_3}. \tag{2}$$

If we denote by $i$ the ratio

$$i = \frac{z_4}{z_3}, \tag{3}$$

then we obtain from relation (2)

$$\omega_3 = \omega_1(1 + i) - i\omega_4. \tag{4}$$

The previous expression gives the link between the angular velocities $\omega_3$, $\omega_4$ and $\omega_1$.

If we denote by $i_{24}$ the ratio

$$i_{24} = \frac{z_4}{z_2}, \tag{5}$$

then from the second expression (1) we obtain the link relation between $\omega_2$, $\omega_1$ and $\omega_4$

$$\omega_2 = \omega_1(1 - i_{24}) + \omega_4 i_{24}. \tag{6}$$

To obtain the Lagrange equations we determine the expression of the mechanical power $P$

$$P = M_1\omega_1 + M_3\omega_3 + M_4\omega_4, \tag{7}$$

where by $M_1$, $M_2$ and $M_3$ were denoted the values of the motor torques for: the thermal engine, the electric machine $MG_1$, and the electric machine $MG_2$, respectively. Keeping into account the relation (4), the expression (7) becomes

$$P = \omega_1\left[M_1 + M_3(1+i)\right] + \omega_4\left(M_4 - iM_3\right). \tag{8}$$

The variation of the elementary work is

$$\partial L = \left[M_1 + M_3(1+i)\right]\partial\theta_1 + \left(M_4 - iM_3\right)\partial\theta_4. \tag{9}$$

Considering that the generalized forces $Q_1$ and $Q_4$ result from the expression of power

$$P = Q_1\dot{\theta}_1 + Q_4\dot{\theta}_4 = \sum_j \left(M_j\omega_j + \vec{F}_j\vec{v}_j\right). \tag{10}$$

where $M_j$, $F_j$ mark the forces and torques that act onto the system, we get the generalized forces $Q_1$ and $Q_4$

$$Q_1 = M_1 + M_3(1+i),\ Q_4 = M_4 - iM_3. \tag{11}$$

For the planetary mechanism the expression of the kinetic energy $T$ is

$$T = \frac{1}{2}\left(J_1\omega_1^2 + J_2\omega_2^2 + m_2R_2^2\omega_1^2 + J_3\omega_3^2 + J\omega_4^2\right), \tag{12}$$

where by $J_1$, $J_2$, $J_3$, $J_4$ we denoted the inertial moments of the elements of mechanism, by $R_2$ the radius where there are the port-satellite pinions 2, while $m_2$ is the mass of these port-satellite pinions. With the notations:

$$A = J_1 + J_2\left(1 - i_{24}\right)^2 + m_2R_2^2 + J_3(1+i)^2,\ B = -J_2 i_{24}\left(1 - i_{24}\right) + J_3 i(1+i),$$
$$C = J_2 i_{24}^2 + J_3 i^2 + J_4, \tag{13}$$

the relation (12) becomes

$$T = \frac{1}{2}\left(A\dot{\theta}_1^2 - 2B\dot{\theta}_1\dot{\theta}_4 + C\dot{\theta}_4^2\right), \tag{14}$$

where $\dot{\theta}_1$ and $\dot{\theta}_4$ stay for the angular velocities $\omega_1$ and $\omega_4$.

To obtain the Lagrange equations

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial\dot{\theta}_k}\right) - \frac{\partial T}{\partial\theta_k} = Q_k,\ k = 1,\ 2, \tag{15}$$

one determine the partial derivatives

$$\frac{\partial T}{\partial\dot{\theta}_1} = A\dot{\theta}_1 - B\dot{\theta}_4,\ \frac{\partial T}{\partial\theta_1} = 0,\ \frac{\partial T}{\partial\dot{\theta}_4} = -B\dot{\theta}_1 + C\dot{\theta}_4,\ \frac{\partial T}{\partial\theta_4} = 0, \tag{16}$$

where $\theta_1$ and $\theta_4$ are the generalized coordinates.

By replacing the previous expressions in the relation (15), one obtains the system of equations

$$A\ddot{\theta}_1 - B\ddot{\theta}_4 = M_1 + M_3(1+i),\ -B\ddot{\theta}_1 + C\ddot{\theta}_4 = M_4 - iM_3, \tag{17}$$

wherefrom results the system of two second order differential equations

$$\ddot{\theta}_1 = \frac{C\left[M_1 + M_3(1+i)\right] + B\left(M_4 - iM_3\right)}{AC - B^2},\ \ddot{\theta}_4 = \frac{A\left(M_4 - iM_3\right) + B\left[M_1 + M_3(1+i)\right]}{AC - B^2}. \tag{18}$$

## THE FUNCTIONING WAYS OF THE HYBRID SYSTEM

In inrush regime from rest, the automotive is acted by the electric machine $MG_2$ (Fig. 2) that works as electric motor. The expression of power given by relation (7) is $P = M_4\omega_4$. The motor $MG_2$ is charged by the inverter $IE$ from din haulage battery, and the thermal engine is stopped ( $\omega_1 = 0$ ). The angular velocity of the electric machine $MG_1$, according to the relation (4) is $\omega_3 = -i\omega_4$; hence the electric machine $MG_1$ rotates in opposite direction to the electric machine $MG_2$.

When a certain speed (50 – 60 km/h) is exceeded, one goes on the thermal engine using the electric machine $MG_1$ as electric motor. The inverter $IE$ charges from the haulage battery $BAT$ the motors $MG_2$ and $MG_1$. Considering a constant angular velocity for the electric machine $MG_2$ of $\omega_2 = 150\,\mathrm{rad/s}$, it results the expression of the angular velocity of getting away for the thermal engine $MT$, $\omega_1 = \dfrac{150 \cdot i + \omega_3}{1 + i}$. If we will consider for $i$ the value $i = 2.3$, and the getting away angular velocity of the thermal engine as $\omega_1 = 150\,\mathrm{rad/s}$, then it will result the angular velocity of the electric machine $MG_1$, $\omega_3 = 150\,\mathrm{rad/s}$.

For the displacement at constant average speed, the automotive is propelled by the thermal engine $MG_1$ and the electric machine $MG_2$. $MG_1$ produces electric energy for the charging of $MG_2$ and, şi eventually, the charging of the haulage battery. The expression of power given by the relation (7) will be $P = M_1\omega_1 - M_3\omega_3 + M_4\omega_4$.

For the displacement at constant high speed, $MG_1$ is blocked, the automobile being propelled by $MT$ and $MG_2$ charging by the haulage battery by the inverter $IE$. Between the two power sources there exists the ratio $\omega_1(1 + i) = i\omega_4$, while the expression of power is $P = M_1\omega_1 + M_4\omega_4$.

At maximum velocity are coupled $MT$, $MG_2$ and $MG_1$, which works as motor and is rotated in opposite direction by the electronic system $IE$. For the same angular velocity of the thermal engine, $MG_2$ will have a greater angular velocity if $MG_1$ is at motor regime and rotated in opposite direction. Both electric machines are charged by the haulage battery $BAT$, the expression of power being $P = M_1\omega_1 + M_3\omega_3 + M_4\omega_4$.

For the recovering brake, $MG_2$ will function as electric generator, the produced energy charges the battery $BAT$. $MT$ is stopped, and $MG_1$ is unplugged and rotates in open circuit with the angular velocity $\omega_3 = -i\omega_4$, hence in opposite direction to the machine $MG_2$.

For the reverse displacement, the motor $MG_2$ rotates in opposite direction to its usual direction, $MT$ is stopped, and $MG_1$ rotates free.

## THE EQUATIONS OF MOTION OF THE HYBRID AUTOMOTIVE

The equations determined in paragraph 5 describe the functioning of the planetary mechanism. To determine the equations of motion of an automotive propelled by such a system of power coupling, we begin again from the expression of mechanical power

$$P = M_1\omega_1 \pm M_3\omega_3 + M_4\omega_4 - P_r - P_a, \tag{19}$$

where by $P_r$ was denoted the power consumed because of the rolling resistance, and by $P_a$ the power consumed because of the air resistance. Their expressions are

$$P_r = F_r \cdot v_a,\ P_a = F_a \cdot v_a, \tag{20}$$

where

$$F_r = f \cdot G_a,\ F_a = \frac{1}{2}\rho c_x A \cdot v_a^2. \tag{21}$$

We used the classical notations: $F_r$ – rolling resistance, $F_a$ – air resistance, $f$ – coefficient of the rolling resistance, $G_a$ – automobile's weight, $\rho$ – air density, $c_x$ – coefficient of air resistance, $A$ –

maximum aria of the cross section of automotive, $v_a$ – displacement velocity of the automotive.

Denoting by $i_0$ the ratio of the principal transmission of automotive and keeping into account the kinematic scheme in Fig. 2, it results the expression of the angular velocity $\omega_r$ of wheel

$$\omega_r = \frac{\omega_4}{i_0 \cdot i_1 \cdot i_2}. \tag{22}$$

We denoted by $i_1$ the transmission ratio of the chain transmission, and by $i_2$ the transmission ration of the gear formatted by the gears $z_5$ and $z_6$

$$i_1 = \frac{r_{P2}}{r_{P1}},\ i_2 = \frac{z_6}{z_5}. \tag{23}$$

Denoting by $r_r$ the rolling radius of the automotive wheel and keeping into account the previous relations, we obtain the expressions of the powers $P_r$ and $P_a$

$$P_r = G_a f \frac{\omega_4}{i_0} r_r,\ P_a = \frac{1}{2}\rho c_x A \frac{\omega_4^3}{i_0^3} r_r^3. \tag{24}$$

Replacing the relations (24) in expression (19), we get

$$P = M_1\omega_1 \pm M_3\omega_3 + M_4\omega_4 - G_a f \frac{\omega_4}{i_0} r_r - \frac{1}{2}\rho c_x A \frac{\omega_4^3}{i_0^3} r_r^3. \tag{25}$$

We used the $\pm$ sign in front of the torque $M_3$ because the electric machine $MG_1$ in Fig. 2 may be generator and then it consumes power, or it may be motor and then it brings power in system.

Keeping into account the relation (4), the expression (25) becomes

$$P = \omega_1\left[M_1 \pm M_3(1+i)\right] + \omega_4\left(M_4 \mp iM_3 - \frac{G_a f r_r}{i_0}\right) - \omega_4^3\left(\frac{1}{2}\rho c_x A \frac{r_r^3}{i_0^3}\right). \tag{26}$$

The variation of the elementary work reads

$$\partial L = \left[M_1 \pm M_3(1+i)\right]\partial\theta_1 + \left(M_4 \mp iM_3 - \frac{G_a f r_r}{i_0} - 3\omega_4^2 \frac{\rho c_x A r_r^3}{2 i_0^3}\right)\partial\theta_4. \tag{27}$$

Hence, the generalized forces $Q_1$ and $Q_4$ have the expressions

$$Q_1 = M_1 + M_3(1+i),\ Q_4 = M_4 \mp iM_3 - \frac{G_a f r_r}{i_0} - 3\omega_4^2 \frac{\rho c_x A r_r^3}{2 i_0^3}. \tag{28}$$

The expression of the kinetic energy $T$ given by the relation (12) is completed by the kinetic energy of the automotive, which, in a simplified model, may be considered consisting in: two front motor wheels with the inertial moments $J_{RF}$, two rear free wheels with the inertial moments $J_{Rs}$, and the automotive of mass $m_a$ having translational motion. In the expressions of the inertial moments of wheels, we also considered the inertial moments of the braking systems, motor axes etc.

$$T = \frac{1}{2}\left(J_1\omega_1^2 + J_2\omega_2^2 + m_2R_2^2\omega_1^2 + J_3\omega_3^2 + J\omega_4^2 + m\left(\frac{\omega_4}{i_0}\right)^2 r_r^2 + 2J_{RF}\left(\frac{\omega_4}{i_0}\right)^2 + 2J_{RS}\left(\frac{\omega_4}{i_0}\right)^2\right). \tag{29}$$

Proceeding analogically to the planetary mechanism, with the notations

$$\begin{gathered} A = J_1 + J_2(1-i_{24})^2 + m_2R_2^2 + J_3(1+i)^2,\ B = -J_2 i_{24}(1-i_{24}) + J_3 i(1+i), \\ C = J_2 i_{24}^2 + J_3 i^2 + J_4 + \frac{m r_r^2}{i_0^2} + \frac{2(J_{RF}+J_{RS})}{i_0^2}, \end{gathered} \tag{30}$$

one obtains the expression of the kinetic energy

$$T = \frac{1}{2}\left(A\dot\theta_1^2 - 2B\dot\theta_1\dot\theta_4 + C\dot\theta_4^2\right), \tag{31}$$

where $\dot\theta_1$ and $\dot\theta_4$ stay for the angular velocities $\omega_1$ and $\omega_4$.

The Lagrange equations given by the expressions (15) read now

$$A\ddot{\theta}_1 - B\ddot{\theta}_4 = M_1 \pm M_3(1+i),\ -B\ddot{\theta}_1 + C\ddot{\theta}_4 = M_4 \mp iM_3 - \frac{G_a f r_r}{i_0} - \frac{3\rho c_x A r_r^3}{2i_0^3}\dot{\theta}_4^2, \tag{32}$$

wherefrom it results the system of two second order differential equations

$$\ddot{\theta}_1 = \frac{C[M_1 \pm M_3(1+i)] + B\left(M_4 \mp iM_3 - \frac{G_a f r_r}{i_0} - \frac{3\rho c_x A r_r^3}{2i_0^3}\dot{\theta}_4^2\right)}{AC - B^2},$$
$$\ddot{\theta}_4 = \frac{A\left(M_4 \mp iM_3 - \frac{G_a f r_r}{i_0} - \frac{3\rho c_x A r_r^3}{2i_0^3}\dot{\theta}_4^2\right) + B[M_1 \pm M_3(1+i)]}{AC - B^2}. \tag{33}$$

Integration of the moving equations (33) is realized numerically, after we firstly established the necessary values to determine the constants.

## CONCLUSIONS

The hybrid automotive is not new, the majority of constructors having such models in the fabrication process. The diversity of the considered solutions made necessary a classification and the establishing of certain mechanical parameters to describe the used systems.

This paper makes a general overview of the coupling systems used in the propelling systems configurations. We presented the principles and general characteristics of the hybrid propelling systems.

Due to the high efficiency, the mechanical systems of power coupling are almost exclusively used to couple the power sources at automotive of small or medium cylindrical capacity.

For the mechanical coupling system used at Toyota Prius configuration, we realized a kinematic analysis and a dynamical one, obtaining the moving equations of the mechanism acted by a thermal engine and two electric machines.

This mechanism is then studied in the compounding of a hybrid automobile, establishing the moving equations for the automotive.

In a future paper we will discuss different numerical situations.

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