DESIGN OPTIMIZATION OF A GEAR SPEED REDUCER BY WAY OF MATLAB & ANSYS

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Abstract: *The preliminary design optimization of spur gear reduction units has been o subject of considerable interest, since many high-performance power transmission applications requires high-performance gear reduction units. One of objectives in the optimal design of single-stage spur gear reduction unit is minimizing the volume. In this paper is used an objective function to formulate the problem of design optimization of gear speed reducer and find an appropriate approach to solve it using Matlab.*

Keywords: stress, strain, objective function, optimization algorithm, safety factor.

INTRODUCTION

Typical engineering systems are described by very larges numbers of variable, and it is the designer task to specify appropriate values for these variables. Skilled designers utilize their knowledge, experience and judgment to specify these variables and design effective engineering systems. Because of the size and complexity of the typical design task, however, even the most skilled designers are unable to take into account all of the variables simultaneously.
Design optimization is the application of numerical algorithms and techniques to engineering systems to assist the designers in improving the system's performance, weight, reliability, cost, etc.
Recently, a variety of computer program have been developed to solve engineering optimization program. Many of this are complex and versatile and the user needs a good understanding of the algorithms/computer programs to be able to use them effectively.
The design of the speed reducer, shown in fig. 1, is considered with the face width b, module of teeth m, number of teeth on pinion z_1, length of shaft 1 between bearings l_1, length of shaft 2 between bearings l_2, diameter of shaft 1 d_1, diameter of shaft 2 d_2,

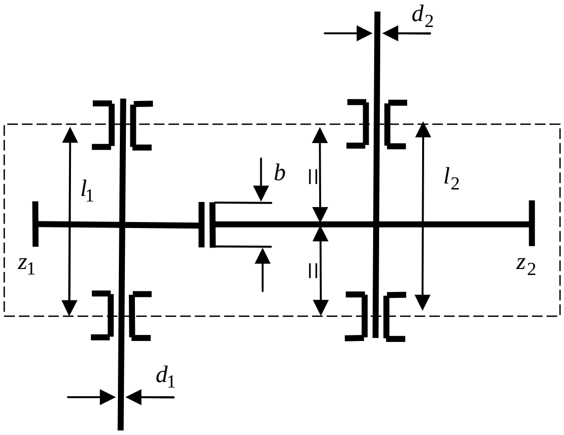

Fig. 1. A single stage spur-gear transmission

MATHEMATICAL MODELLING

The bending stress at the root of the tooth is given by the following simplified formula

$$\sigma_F = const_F \cdot \frac{F_t}{bm} = const \cdot \frac{2T_1}{bm^2 z_1} \tag{1}$$

where

σ_F - bending stress at the root of the tooth

$const_F$ - combination of all constant or approximately constant occurring in ISO or AGMA formulae.

F_t - tangential load at pitch circle

b - face width

z_1 - number of teeth on pinion

m - module of gear teeth

T_1 - input torque at shaft 1.

The contact stress is given by the following, also simplified formula

$$\sigma_H = const_H \cdot \sqrt{\frac{F_t}{bmz_1} \cdot \frac{u+1}{u}} = const_H \cdot \sqrt{\frac{2T_1}{bm^2 z_1^2} \cdot \frac{u+1}{u}} \tag{2}$$

where

$const_H$ - combination of all constant or approximately constant factors in the ISO or AGMA formulae

u - gear ratio $(u \geq 1)$.

For steels case hardened to $HRC = 60$, the permissible bending stress is $\sigma_{p\,\mathbf{lim}} = 500MPa$ and the permissible contact stress is $\sigma_{H\,\mathbf{lim}} = 1600MPa$ (according to MAAG).

The maximum linear deflections of the shafts 1 and 2 are given by relations

$$y_1 \approx \frac{F_t l_1^3}{48EI_1} = \frac{128}{48\pi} \cdot \frac{T_1 l_1^3}{Ed_1^4 mz_1} \tag{3}$$

$$y_2 \approx \frac{F_t l_2^3}{48EI_2} = \frac{128}{48\pi} \cdot \frac{T_1 l_2^3}{Ed_2^4 mz_1} \tag{4}$$

where d_1, d_2 are diameters of input and output shafts 1, respective 2.

Allowable linear deflections will depend on many factors and for this case are considered as form $y_{all1,2} = 3l_{1,2} \cdot 10^{-4}$ for steel with modulus of elasticity $E = 2 \cdot 10^5 MPa$.

Assuming a solid shafts with round sections, the fluctuating stresses due to bending and torsion are given by

$$\sigma_{1,2} = \frac{32M_{1,2}}{\pi d_{1,2}^3}\,;\, \tau_{1,2} = \frac{16T_{1,2}}{\pi d_{1,2}^3} \tag{5}$$

where $T_{1,2}$ and $M_{1,2}$ are the torque, respective bending moment at shafts 1 and 2.

Combining these stresses in accordance with the distortion energy failure theory, the von Mises stresses for rotating shafts, neglecting axial loads, are given by

$$\sigma_{e1,2} = \sqrt{\sigma_{1,2}^2 + 3\tau_{1,2}^2} = \frac{16}{\pi d_{1,2}^3}\sqrt{4M_{1,2}^2 + 3T_{1,2}^2} \tag{6}$$

Since

$$M_1 = \frac{F_t}{2} \cdot \frac{l_1}{2} = \frac{T_1}{mz_1} \cdot \frac{l_1}{2} \tag{7}$$

$$M_2 = \frac{F_t}{2} \cdot \frac{l_2}{2} = \frac{T_1}{mz_1} \cdot \frac{l_2}{2} \tag{8}$$

relations (6) can be written as

$$\sigma_{e1} = \frac{16T_1}{\pi d_1^3}\sqrt{\frac{l_1^2}{m^2 z_1^2} + 3} \tag{9}$$

$$\sigma_{e2} = \frac{16T_1}{\pi d_2^3}\sqrt{\frac{l_2^2}{m^2 z_1^2} + 3u^2} \quad (10)$$

In case of use the same material for pinion, shat and gear, the weight of speed reducer is give by simplified relation

$$W = \frac{\pi}{4}\rho\left\{d_1^2 l_1 + d_2^2 l_2 + b\left(m^2 z_1^2 - d_1^2\right) - b\left[m^2 (u z_1)^2 - d_2^2\right]\right\} \quad (11)$$

where ρ is the density of material.

In case of use a set of design parameters $\mathbf{x} = [x_1, x_2, x_3, \ldots, x_7]^T$, defined as: $x_1 = b$; $x_2 = m$; $x_3 = z_1$; $x_4 = l_1$; $x_5 = l_2$; $x_6 = d_1$; $x_7 = d_2$, the objective function (minimization of weight of speed reducer) can be written as form

$$f(x) = \frac{\pi}{4}\left[x_6^2 x_4 + x_7^2 x_5 + x_1 x_2^2 x_3^2\left(1 + u^2\right) - x_1\left(x_6^2 + x_7^2\right)\right] \quad (12)$$

The constraints include limitations on the bending stress of gear teeth, surface stress, transverse deflections of shafts 1 and 2 due to transmitted force, and stresses in shafts 1 and 2. In concordance with these limitations, the nonlinear constraints are given by relations

$$const_H \cdot \left[2T_1 x_1^{-1} x_2^{-2} x_3^{-3}(u+1)u^{-1}\right]^{0.5} - 1600 \le 0 \quad (13)$$

$$const_F \cdot \left(2T_1 x_1^{-1} x_2^{-2} x_3^{-1}\right) - 500 \le 0 \quad (14)$$

$$\frac{128}{48\pi E} T_1 x_2^{-1} x_3^{-1} x_4^3 x_6^4 - 3x_4 \cdot 10^{-4} \le 0 \quad (15)$$

$$\frac{128}{48\pi E} T_1 x_2^{-1} x_3^{-1} x_5^3 x_7^4 - 3x_5 \cdot 10^{-4} \le 0 \quad (16)$$

$$\frac{16T_1}{\pi} x_6^{-3}\left(x_4^2 x_2^{-2} x_3^{-2} + 3\right)^{0.5} - 100 \le 0 \quad (17)$$

$$\frac{16T_1}{\pi} x_7^{-3}\left(x_2^{-2} x_3^{-2} x_5^2 + 3u^2\right)^{0.5} - 100 \le 0 \quad (18)$$

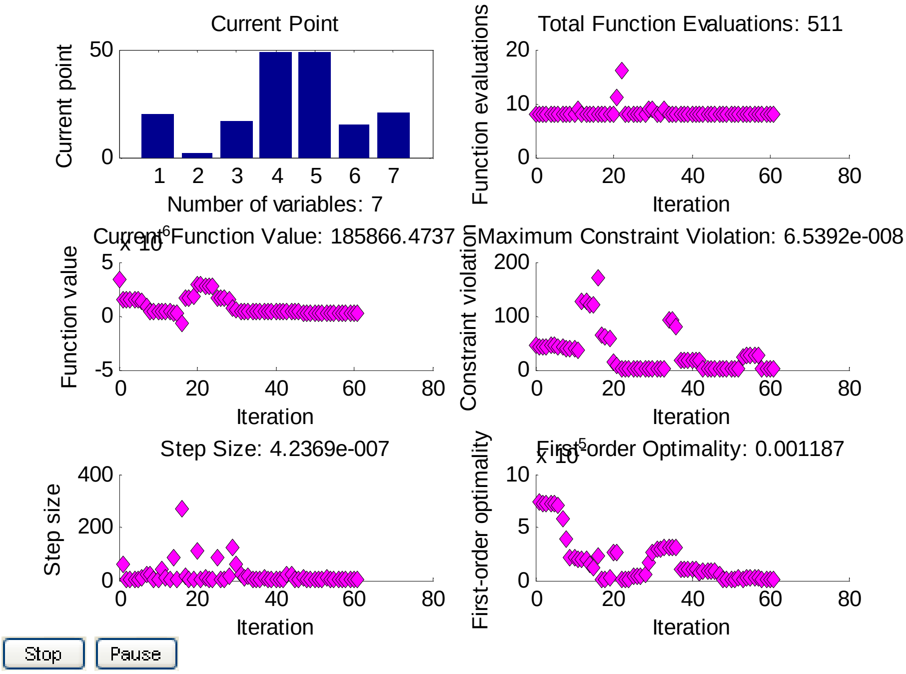

Fig. 2. Optimization plot function by Matlab/Optimization tool

For the following parameters: $u = 3$; $T_1 = 32703[Nmm]$; $const_H = 629.473$; $const_F = 9.75$, the optimization problem is solved by the *fmincon* function using Matlab/Optimization tool.

The solution is:

$$x = [29.5764; 1.9576; 17.0000; 48.9412; 48.9412; 15.5834; 20.7965]$$

where: $b = 29.5764mm$; $m = 1.9576mm$; $z_1 = 17$; $l_1 = 68.9412$; $l_2 = 68.9412$; $d_1 = 15.5834$; $d_2 = 20.7965$.

The optimization plot functions by *Matlab/optimization tool* is indicated in figure 2.

SIMULATION BY ANSYS

A proposal sketch of output shaft is presented in figure 3 (dash line for simplified shape and solid line for real shape)

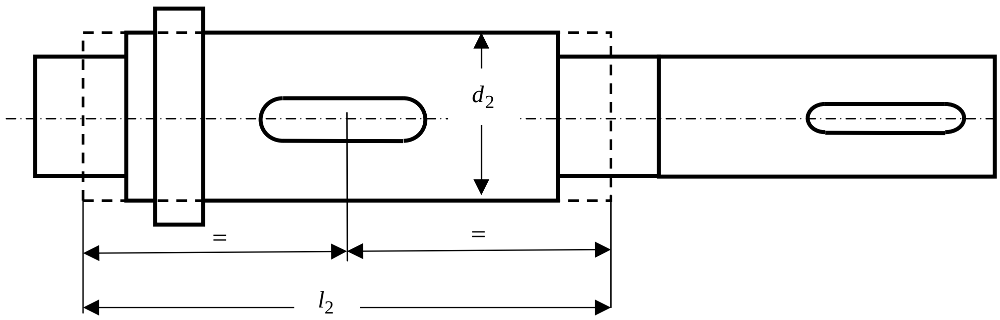

Fig. 3. Output shaft of speed reducer

Equivalent stress (also called von Mises stress) is often used in design work because it allows any arbitrary three-dimensional stress state to be represented as a single positive stress value. Equivalent stress (figure 4) is related to the principal stresses $(\sigma_1 > \sigma_2 > \sigma_3)$, by the equation

$$\sigma_e = \sqrt{\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}} \quad (19)$$

The maximum normal stress (called the principal stress σ_1) and the minimum normal stress (called the principal stress σ_2), given by equations

$$\sigma_1 = \frac{\sigma_{xx} + \sigma_{yy}}{2} + \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2} \quad (20)$$

$$\sigma_2 = \frac{\sigma_{xx} + \sigma_{yy}}{2} - \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2} \quad (21)$$

The maximum shear stress $(\tau_{\mathbf{max}})$ and the minimum shear stress $(\tau_{\mathbf{min}})$ given by equations

$$\tau_{\mathbf{max}} = \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2} \quad (22)$$

$$\tau_{\mathbf{min}} = -\sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2} = -\tau_{\mathbf{max}} \quad (23)$$

The von Mises or equivalent strain is computed as

$$\varepsilon_e = \frac{1}{1 + \upsilon'}\sqrt{(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2} \quad (24)$$

where ε_1, ε_2, ε_3 are the principal strains $(\varepsilon_1 > \varepsilon_2 > \varepsilon_3)$ and υ' is effective Poisson's ratio.

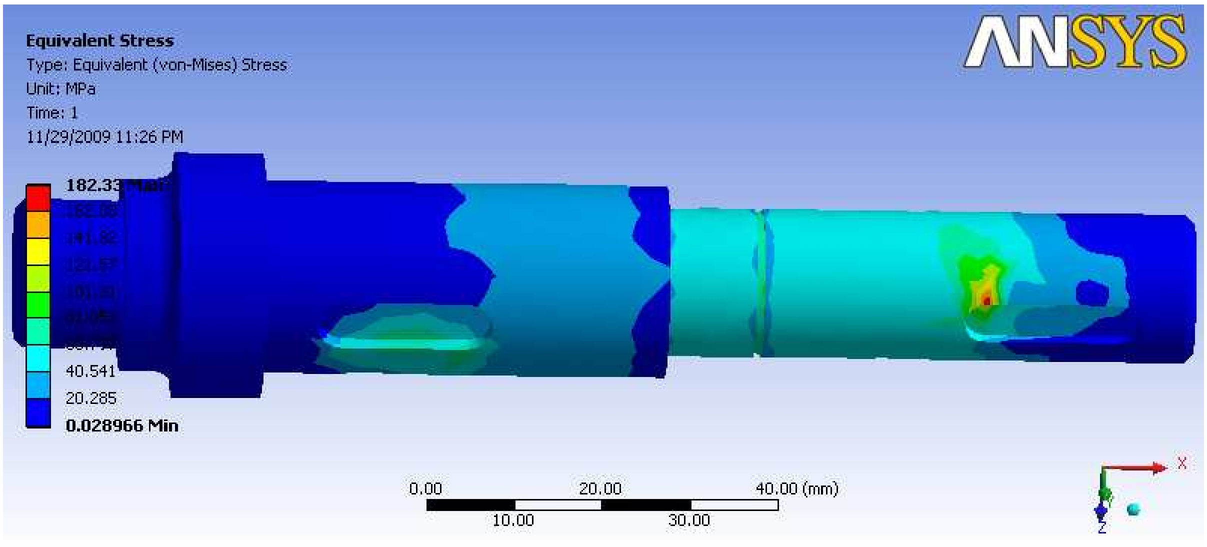

Fig. 4. Equivalent stress

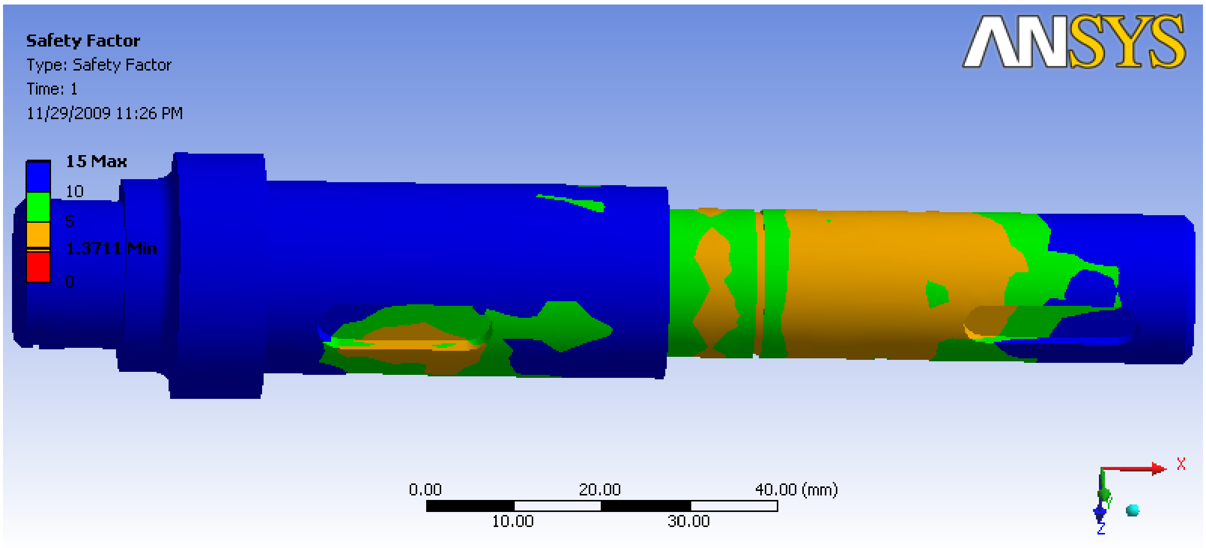

Fig. 5. Safety factor (maximum equivalent stress failure theory)

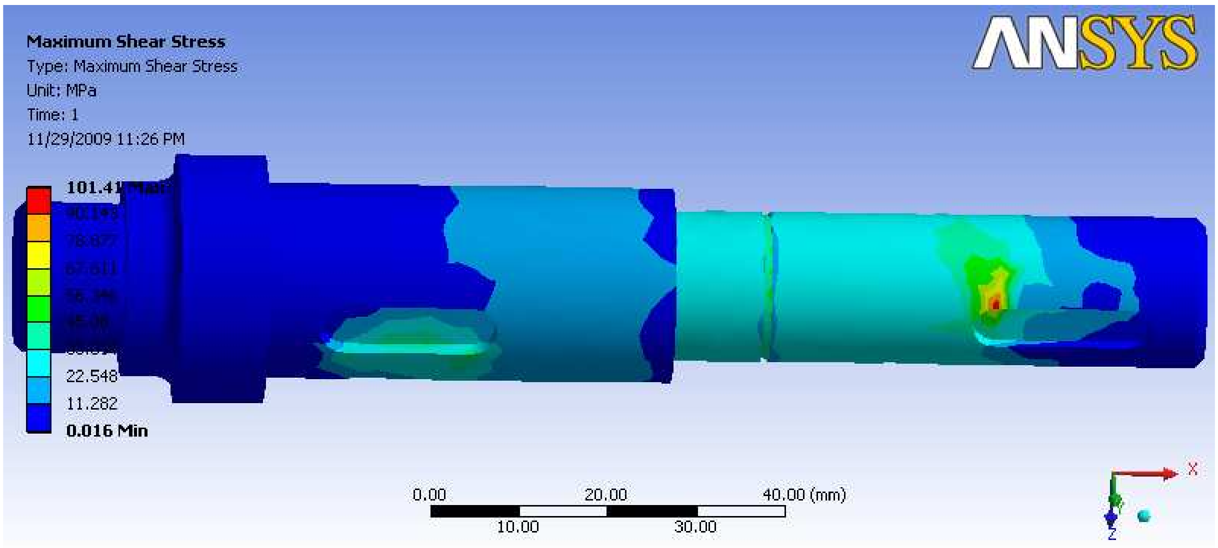

Fig. 6. Maximum shear stress

The maximum equivalent stress failure theory is generally considered as the most appropriate for ductile materials such as aluminum, brass and steel.
Based on this theory, a particular combination of principal stresses causes failure if the maximum equivalent stress in a structure equals or exceeds a specific stress limit

$$\sigma_e \geq S_{\text{lim}} \tag{25}$$

Expressing the theory as a design goal

$$\frac{\sigma_e}{S_{\text{lim}}} < 1 \tag{26}$$

and safety factor (figure 5) is defined as

$$F_s = \frac{S_{\text{lim}}}{\sigma_e} \tag{27}$$

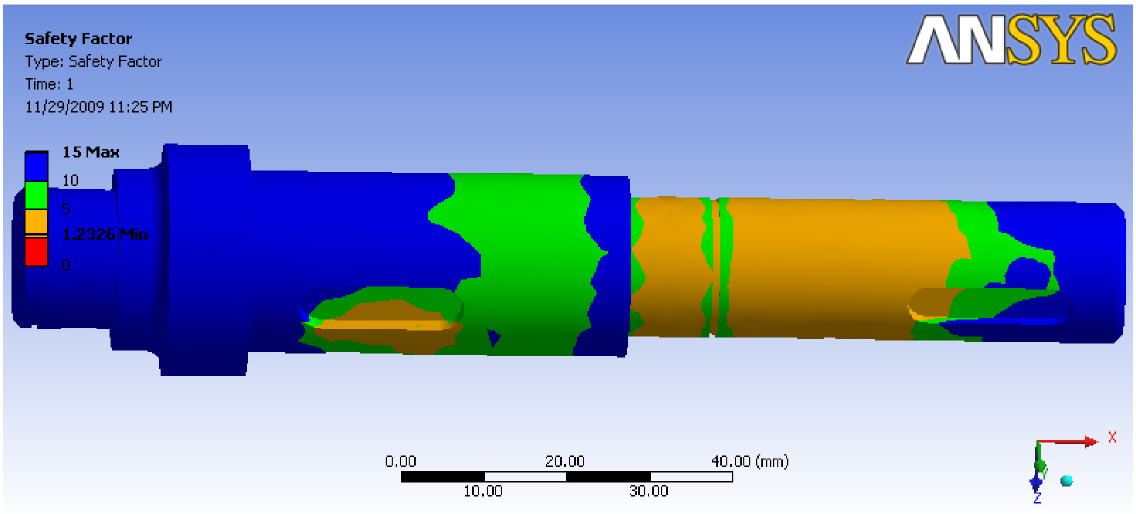

Fig. 7. Safety factor (maximum shear stress failure theory)

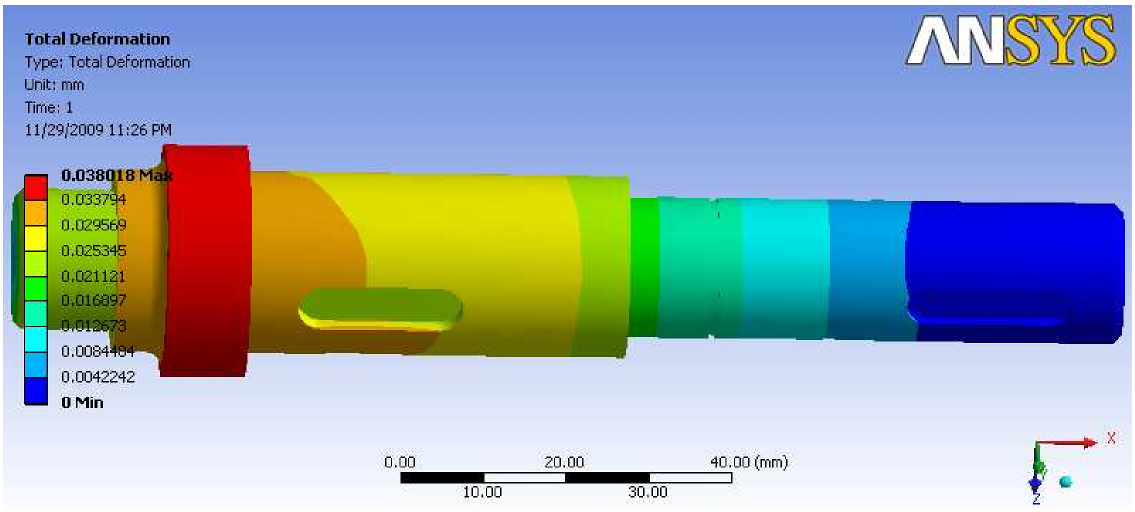

Fig. 8. Total deformation

Based on the maximum shear stress failure theory, a particular combination of principal stresses causes failure if the maximum shear equals or exceeds a specific shear limit

$$\tau_{\text{max}} \geq f \cdot S_{\text{lim}} \tag{28}$$

where the limit strength is generally the yield or ultimate strength of the material. The shear strength of the material is defined as a fraction $(f < 1)$ of the yield or ultimate strength.

Based on this theory, the safety factor (figure 7) is defined as

$$F_s = \frac{f \cdot S_{\lim}}{\tau_{\max}} \qquad (29)$$

Physical deformations are calculated relative to the fixed Cartesian (X, Y, Z) coordinate system defined for a output-shaft by the CAD system.

Total deformation (figure 8) is

$$U = \sqrt{U_x^2 + U_y^2 + U_z^2} \qquad (30)$$

where U_x, U_y, U_z are the component deformations with respect to X, Y and Z directions.

CONCLUSIONS

This paper presented a simplified model of optimal calculus of a speed reducer with a single stage of reduction. With the aid of Matlab are determined optimal dimensions for a simplified constructive scheme of a speed reducer with minimum weight but strong enough to principal mechanical stresses. Are considered the bending stress at the root and the contact stress of teeth, the linear deflections of the shafts and the von Mises stresses of rotating shafts within there allowable limits.

By means of ansys are highlighted the von Mises and the maximum shear stresses, the safety factors (based on the maximum equivalent stress failure theory respective on the maximum shear stress failure theory), total deformation of the output shaft. With same specific modifications, this optimization model can be extended to other mechanical systems

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