UNIVERSITY OF PITESTI SCIENTIFIC BULLETIN
FACULTY OF MECHANICS AND TECHNOLOGY AUTOMOTIVE series, year XV, no.19, vol. A

SIMULATION IN COMMON INTERNAL COMBUSTION ENGINE AND HYDRAULIC TRANSFORMER IN CASE OF CONTINOUS TRANSMISSION

Salvadore Mugurel BURCIU[1]
[1]"Lower Danube" University of Galati, Romania, Faculty of Mechanical Engineering , Department of Thermal and Environmental Engineering Systems, email: Mugurel.Burciu@ugal.ro

Abstract: *The paper deals with the mathematical model of the in common unsteady and steady working conditions of hydraulic transformer and internal combustion engine . For the unsteady and steady in common working conditions , the recommended system of differential equations is possible to solve using numerical methods and permits to find the unknown variables which are displacement speed , angular speed of hydraulic pump and ratio of change , depending on known parameters and time .The paper presents some graphical results that are obtained with a program written in Mathlab, for a numerical case considered.*

Keywords: vehicle, modeling, simulation, internal combustion engine, continous transmission

INTRODUCTION

Paper presents the mathematical model of the unsteady and steady in common working conditions of hydraulic transformer and internal combustion engine. The mathematical model includes the differential equations for unsteady and steady working conditions, specify the parameters known and unknown variables can be calculated using numerical methods. Paper contains some graphic results that are obtained for a numerical case considered.

MATHEMATICAL MODEL

The general equation of motion,[1], [4], in case of continuously variable transmission is:

$$\frac{dv}{dt} = \frac{1}{m_a \cdot \delta_c} \cdot \left(F_{tc} - \sum R \right) \tag{1}$$

where:

$$F_{tc} = \frac{M_T \eta'_{tr} \cdot i'_{tr}}{r_r} = \frac{M_P \cdot k \cdot \eta'_{tr} \cdot i'_{tr}}{r_r} \tag{2}$$

represents the mechanical force of traction , determined by hydraulic turbine torque of hydraulic transformer ; η'_{tr}, i'_{tr} are efficiency and the transmission ratio of mechanical transmission behind the hydraulic converter ; δ_c is the reduction coefficient of mass which is found in translation and rotary movement , mass which is reduced to the shaft of propulsion wheels; the coefficient δ_c depends on ratio for transforming ' i ' of hydraulic transmission .
The sum of movement resistance forces ,[1], for example in case of truck with ' n ' tows is:

$$\sum R = \left(G_a + \sum_1^n G_{rem} \right) \cdot \left(f \cdot \cos\alpha + \sin\alpha \right) + \frac{k \cdot A \cdot v^2}{13} + \sum_1^n \frac{k' \cdot A_{rem} \cdot v^2}{13} \tag{3}$$

The sum of movement resistance forces in case of truck without tows:

$$\sum R = G_a \cdot \left(f \cdot \cos\alpha + \sin\alpha \right) + \frac{k \cdot A \cdot v^2}{13} \tag{4}$$

The torque of total movement resistances, reduced to the shaft of hydraulic turbine:

$$M_{Rt} = \frac{G_a\left(f\cos\alpha + \sin\alpha\right)}{\eta'_{tr}\cdot i'_{tr}}\cdot r_r + \frac{kA\cdot v^2}{13\cdot\eta'_{tr}\cdot i'_{tr}}\cdot r_r = \frac{G_a\left(f\cos\alpha + \sin\alpha\right)}{\eta'_{tr}\cdot i'_{tr}}\cdot r_r + \frac{kA\cdot r_r^3 n_T^2}{13\cdot\eta'_{tr}\cdot i'^3_{tr}}\cdot\left(\frac{3{,}6\pi}{30}\right)^2 \tag{5}$$

The kinematic, dynamic and efficiency parameters which makes the relation between the entrance parameters of pump and exit parameters of turbine ,[1], [2], [3], are:

-- ratio of transmission:

$$i_{tr} = \omega_1/\omega_2 = \omega_P/\omega_T = n_1/n_2 = n_P/n_T \tag{6}$$

-- ratio of change (transforming):

$$i = n_2/n_1 = n_T/n_P = 1/i_{tr} \tag{7}$$

-- sliding:

$$a = \left(n_1 - n_2\right)/n_1 = \left(n_P - n_T\right)/n_P = 1 - i = 1 - 1/i_{tr} \tag{8}$$

-- the torque at the shaft of hydraulic pump:

$$M_P = \lambda_P\cdot\rho\cdot D^5 n_P^2 = \lambda_1\cdot\rho\cdot D^5 n_1^2 \tag{9}$$

-- the torque at the shaft of hydraulic turbine:

$$M_T = \lambda_T\rho\cdot D^5 n_P^2 = \lambda_2\rho\cdot D^5 n_1^2 \tag{10}$$

-- the amplification coefficient:

$$k = \left|M_T\right|/M_P = \lambda_T/\lambda_P = \lambda_2/\lambda_1 \tag{11}$$

-- hydraulic transformer efficiency:

$$\eta = \frac{P_T}{P_P} = \frac{M_T\cdot\omega_2}{M_P\cdot\omega_1} = k\cdot i \tag{12}$$

The figure 1 presents the scheme of a continous variable transmission with hydraulic transformer.

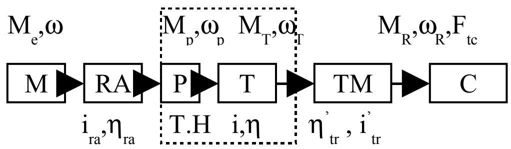

Fig. 1

The elements specified are: M – internal combustion engines ; R.A – speed reducing adapting gear; P, T-hydraulic pump respectively hydraulic turbine of hydraulic transformer; T.H – hydraulic transformer; TM – mechanical transmission ; C – consumer ;

The model presented in figure 1 can be reduced in two equivalent masses which are hydraulical coupled through the hydraulic transformer: - one mass which includes the masses in rotary motion of engine and masses due to shaft of hydraulic pump , all reduced to the shaft of pump ;
- the other mass which includes the masses in rotary and translation motion of truck and mechanical transmission behind hydraulic transformer, the masses due to shaft of turbine, all reduced to the shaft of hydraulic turbine.

The differential equation which simulate the dynamical motion of first inertial group is:

$$M_e\cdot i_{ra}\eta_{ra} - M_P = I_1\frac{d\omega_P}{dt} \tag{13}$$

where the inertial moment I_1 for the first inertial group is:

$$I_1 = I_P + I_m\cdot i_{ra}^2 \tag{14}$$

and $i_{ra} = \omega_m/\omega_p$, ratio of transmission for the speed reducing adopting gear ; was ignored the small influence of adopting gear.

The differential equation which simulate the dynamical motion of second inertial group is:

$$M_T - \frac{F_{tc}r_r}{i'_{tr}\eta'_{tr}} = I'_2\frac{d\omega_T}{dt} \tag{15}$$

where the inertial moment I_2' of second inertial group is:

$$I_2' = I_T + \frac{I_{tr'}}{i_{tr'}^2} + \frac{\sum I_r}{i_{tr'}^2} = I_2 + \frac{\sum I_r}{i_{tr'}^2} \tag{16}$$

$i'_{tr} = \omega_t / \omega_r$, ratio of mechanical transmission ; $I_{tr'}$ – inertial moment of masses from mechanical transmission , masses which are in rotary motion ; I_r – inertial moment from a propulsion wheel. With a view to calculate the reduced total mass we write:

$$m_{red} \cdot \frac{dv}{dt} = I_2 \cdot \frac{d\omega_T}{dt} \cdot \frac{\eta'_{tr} \cdot i'_{tr}}{r_r} + \sum \frac{I_r}{r_r} \cdot \frac{d\omega_r}{dt} + m_a \frac{dv}{dt}; \quad \frac{d\omega_T}{dt} = \frac{dv}{dt} \cdot \frac{i'_{tr}}{r_r}; \quad \frac{d\omega_r}{dt} = \frac{dv}{r_r dt}$$

Results:

$$m_{red} = m_a + I_2 \cdot \frac{i'^2_{tr}}{r_r^2} \cdot \eta'_{tr} + \sum I_r \cdot \frac{1}{r_r^2} \tag{17}$$

Mechanical propulsion force is:

$$F_{tc} = M_T \cdot \frac{i'_{tr} \cdot \eta'_{tr}}{r_r} = k \cdot M_P \cdot \frac{i'_{tr} \cdot \eta'_{tr}}{r_r} = k \cdot \frac{i'_{tr} \cdot \eta'_{tr}}{r_r} \left(M_e \cdot i_{ra} \eta_{ra} - I_1 \frac{d\omega_P}{dt} \right) \tag{18}$$

Because:

$$\frac{d\omega_P}{dt} = \frac{i'_{tr}}{r_r} \left(i_{tr} + v \frac{di_{tr}}{dv} \right) \cdot \frac{dv}{dt} \tag{19}$$

Results:

$$F_{tc} = k \cdot \frac{i'_{tr} \cdot \eta'_{tr}}{r_r} \left(M_e \cdot i_{ra} \eta_{ra} - I_1 \frac{i'_{tr}}{r_r} \left(i_{tr} + v \frac{di_{tr}}{dv} \right) \cdot \frac{dv}{dt} \right) \tag{20}$$

If substitute relations (17) and (20) into relation (1) results:

$$\frac{dv}{dt} \left\{ m_a + \frac{i'^2_{tr}}{r_r^2} \cdot \eta'_{tr} \left[I_2 + I_1 \left(i_{tr} + v \frac{di_{tr}}{dv} \right) k \right] + \sum I_r \cdot \frac{1}{r_r^2} \right\} = M_e \cdot i_{ra} \eta_{ra} \cdot k \cdot \frac{i'_{tr} \cdot \eta'_{tr}}{r_r} - \sum R \tag{21}$$

Using the identification of mathematical terms between relations (1) and (21) results the reduction coefficient of mass in case of continuously variable transmission:

$$\delta_c = 1 + \frac{1}{m_a} \cdot \frac{i'^2_{tr}}{r_r^2} \cdot \eta'_{tr} \left[I_2 + I_1 \left(i_{tr} + v \frac{di_{tr}}{dv} \right) k \right] + \frac{1}{m_a} \cdot \sum I_r \cdot \frac{1}{r_r^2} \tag{22}$$

Respectively force of traction:

$$F_{tc} = M_e \cdot i_{ra} \eta_{ra} \cdot k \cdot \frac{i'_{tr} \cdot \eta'_{tr}}{r_r} \tag{23}$$

where the sum of movement resistance forces , for example in case of truck with ‘ n ‘ tows , is:

$$\sum R = \left(G_a + \sum_1^n G_{rem} \right) \cdot (f \cdot \cos\alpha + \sin\alpha) + \frac{k \cdot A \cdot v^2}{13} + \sum_1^n \frac{k' \cdot A_{rem} \cdot v^2}{13} \tag{24}$$

Relations (13), (15) and (21), represent the mathematical model for the dynamical movement of propulsion with continuously variable transmission:

$$\begin{aligned} & M_e(\omega_p \cdot i_{ra}, \chi_s) \cdot i_{ra} \eta_{ra} - M_P(\omega_p, i) = I_1 \frac{d\omega_P}{dt} \\ & M_P(\omega_P, i) \cdot k(\omega_p, i) - \frac{F_{tc}(i, v) \cdot r_r}{i'_{tr} \eta'_{tr}} = I'_2 \frac{d\omega_T}{dt} \end{aligned} \tag{25}$$

$$\frac{dv}{dt} \left\{ m_a + \frac{i'^2_{tr}}{r_r^2} \cdot \eta'_{tr} \left[I_2 + I_1 \left(i_{tr} + v \frac{di_{tr}}{dv} \right) k(\omega_p, i) \right] + \sum I_r \cdot \frac{1}{r_r^2} \right\} = M_e(\omega_p \cdot i_{ra}, \chi_s) \cdot i_{rd} \eta_{ra} \cdot k(\omega_p, i) \cdot \frac{i'_{tr} \cdot \eta'_{tr}}{r_r} -$$

$$- G_a \cdot (f \cdot \cos\alpha + \sin\alpha) - \frac{k \cdot A \cdot v^2}{13}$$

The unknown variables from system of equations (25) are: speed of movement ' v ' , angular speed of hydraulic pump ' ω_p ' and ratio of transforming ' i ' , where the angular speed of hydraulic turbine is $\omega_T = i \cdot \omega_p$.

The known variables are : the torque of engine $M_e(n = n_p \cdot i_{ra}, \chi_s)$; i_{ra} , η_{ra} ratio of transmission respectively the efficiency of speed reducing adopting gear ; χ_s coefficient of load ;

I_1 , I'_2 inertial moments for the first and second inertial group specified before ; k , i'_{tr} , η'_{tr} are specified before ; r_r radius of rolling for the propulsion wheels .

The system of equations (25) can be solve using numerical methods .

For a specified hydraulic transformer in case of continuous variable transmission , the force of traction can be represented as a function depends on , displacement speed ' v [km/h]' , speed of revolution for pump ' n_p[r.p.m] ' or speed of revolution for turbine ' n_T[r.p.m] ', accordance with:

$$F_{tc} = kM_p \eta'_{tr} i'_{tr} / r_r = kM_e \cdot i_{ra} \eta_{ra} \cdot \eta'_{tr} i'_{tr} / r_r \tag{26}$$

Because :

$$M_p = \rho \cdot \lambda_p D^5 n_p^2 \ ; \ n_p / n_T = i_{tr} ; n_T / n_r = i'_{tr} \ ; \ n_r = 2{,}65 \frac{v}{r_r} \tag{27}$$

Results the following expressions for the traction force :

$$F_{tc} = k \cdot \rho \cdot \lambda_p D^5 n_p^2 \cdot \eta'_{tr} i'_{tr} / r_r = f(n_p) \tag{28}$$

$$F_{tc} = k \cdot \rho \cdot \lambda_p D^5 n_T^2 \cdot i_{tr}^2 \cdot \eta'_{tr} i'_{tr} / r_r = f(n_T) \tag{29}$$

$$F_{tc} = k \cdot \rho \cdot \lambda_p D^5 n_r^2 \cdot i_{tr}^2 \cdot \eta'_{tr} \cdot i'^3_{tr} / r_r = f(n_r) \tag{30}$$

$$F_{tc} = 2{,}65^2 \cdot k \cdot \rho \cdot \lambda_p D^5 \cdot i_{tr}^2 \cdot \eta'_{tr} \cdot i'^3_{tr} \cdot v^2 / r_r^3 = f(v) \tag{31}$$

Usually is preferably to represent the traction force as a function depends of movement speed v[km/h] or depends of revolution speed of turbine n_T[r.p.m] (relations 31 respectively 29) .

Because we can write:

$$i_{tr} + v \cdot \frac{di_{tr}}{dv} = \frac{d\omega_p}{dt} \cdot \frac{r_r}{i'_{tr}} \cdot \frac{1}{dv/dt} \tag{32}$$

using the third equation and the first equation of system (25) , results:

$$\frac{dv}{dt} \left\{ m_a + \frac{i'^2_{tr}}{r_r^2} \cdot \eta'_{tr} \cdot I_2 + \sum I_r \cdot \frac{1}{r_r^2} \right\} = M_p(\omega_p, i) \cdot k(\omega_p, i) \cdot \frac{i'_{tr} \cdot \eta'_{tr}}{r_r} - G_a \cdot (f \cdot \cos\alpha + \sin\alpha) - \frac{k \cdot A \cdot v^2}{13} \tag{33}$$

If we consider:

$$A = m_a + \frac{i'^2_{tr}}{r_r^2} \cdot \eta'_{tr} \cdot I_2 + \sum I_r \cdot \frac{1}{r_r^2} \ ; \ B = \frac{i'_{tr} \cdot \eta'_{tr}}{r_r} \ ; \ C = G_a \cdot (f \cdot \cos\alpha + \sin\alpha) \ ; \ D = \frac{k \cdot A}{13} \tag{34}$$

Results the following differential equation [1]:

$$\frac{dv}{dt} = \frac{M_p \cdot k \cdot B}{A} - \frac{C}{A} - \frac{D}{A} \cdot v^2 \tag{35}$$

RESULTS

If we do not consider reducing adapting gear and $\eta = k \cdot i \approx \text{constant}$, knowing the following variation $M_p = M_p(\omega_p = \omega_{engine}, i)$, we get (using equation 35) the diagrams: $v = v(t, n_p = n_{engine}, i = const)$ and $F_{tc} = F_{tc}(t, n_p = n_{engine}, i = const)$, for a numerical example, [1], also using a personal computer program written in Mathlab.

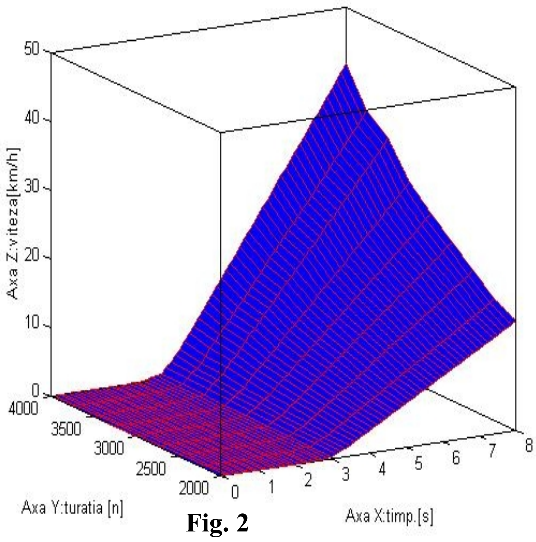

Fig. 2

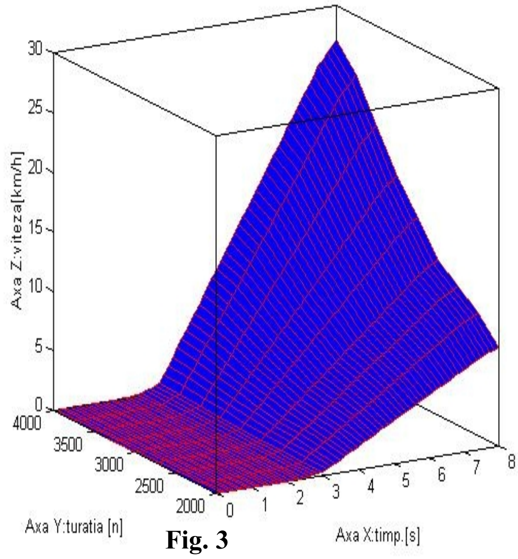

Fig. 3

$$v[km/h] = v(t, n_p = n_{engine}, i = 0.3) \quad \text{for Figure 2}$$

$$v[km/h] = v(t, n_p = n_{engine}, i = 0.4) \quad \text{for Figure 3}$$

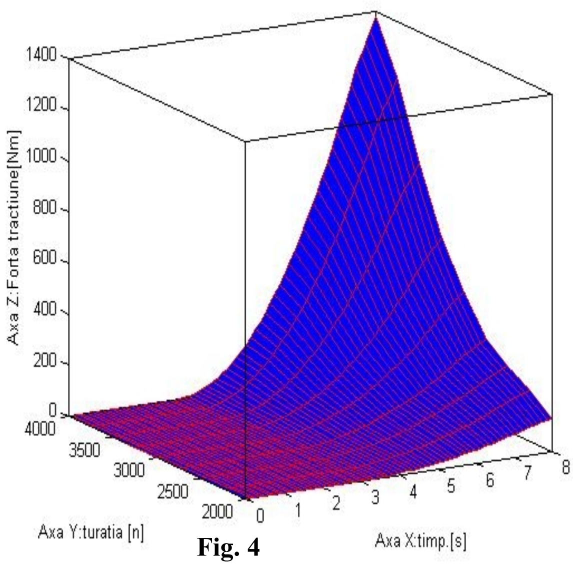

Fig. 4

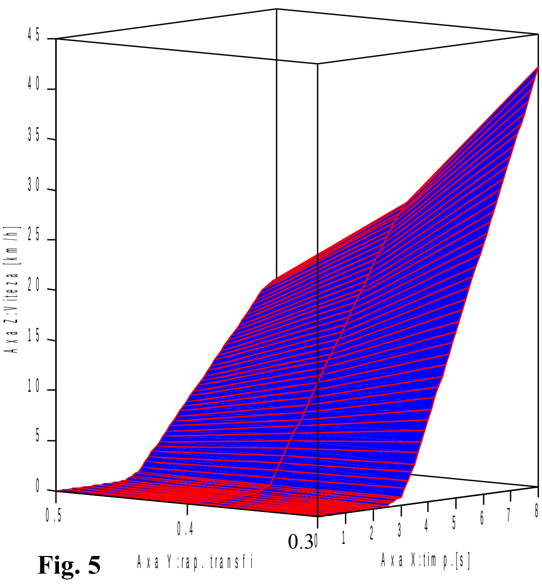

Fig. 5

$$F_{tc}[N] = F_{tc}(t, n_p = n_{engine}, i = 0.4) \quad \text{for Figure 4}$$

$$v[km/h] = v(t, i \in [0{,}3;0{,}5], n = 4000[rpm]) \quad \text{for Figure 5}$$

REFERENCES

[1] Burciu S.M., *Propulsion installations with internal combustion engines*, Publisher Bren, Bucharest 2003.

[2] Năstăsoiu S., *Turbomachines and drive motor vehicle*, University of Brasov , 1980.

[3] Peligrad N., *Hydraulic couplings and hydraulic torque converters*, Technical Publishing, Bucharest ,1985.

[4] Ciolan Ghe., Preda I., Pereş Ghe., *Gear boxes for automobile*, Didactic and Pedagogical Publishing, Bucharest 1998.