UNIVERSITY OF PITESTI *SCIENTIFIC BULLETIN*
FACULTY OF MECHANICS AND TECHNOLOGY **AUTOMOTIVE series, year XV, no.19, vol. B**

ASPECTS OF THE VERTICAL FORCES DISTRIBUTION AT THE MOTOR VEHICLES

Victor OTAT, Dumitru BOLCU, Marius M. STANESCU
University of Craiova, Faculty of Mechanics, Romania

Abstract: *The number of unknowns is more numerous than the number of equations in the motion equations of a motor vehicle. For that reason to determine the distribution of the contact forces with the road there are necessary additional suppositions. One of these suppositions takes into account the neglect of the car body's strain. Under the circumstances it is considered that the vertical forces are proportional to the displacements on vertically of the suspension fixing points.*

Consequently there are determined the vertical forces over each vehicle wheel. There are presented the dependences of these ones depending on: the longitudinal acceleration, the speed and the angular acceleration of the motor vehicle, the mass and the additional weight and the distribution of this one by the coordinates compared to the theoretical centre of the road vehicle.

These are particularized the relations obtained for the passing over from the rectilinear motion to the circular motion, presenting the variations of the vertical forces depending on the angle between the speed of the mass centre and the longitudinal axis of the vehicle. There are also determined the limit values of these forces taking into account that the motion of the mass centre is circular. Therefore it is observed a burden of the vehicle's exterior part, overloaded being the front wheel exterior to the bend. At the same time it is observed a discharged especially at the back wheel interior to the bend. It is also determined the critical velocity at which the vertical reactions at the back interior wheel becomes null, so appearing the danger of the side slipping.

Keywords: vertical forces, non steady motion

1. INTRODUCTION

In the cars moving equations, the number of unknown parameters is bigger then the number of moving equations. For this reason, the simplified hypothesis are necessary in order to determinate the distribution of forces from contact area between wheels and the road. One of those hypotheses is referring to neglect the body deformation relative to suspension deformations. In those conditions, it is considerate that the vertical forces are proportional with the deformations and the deformation velocities of the suspensions.

In this paper the vertical forces are determinate in case of transition between the rectilinear movement and the circular movement. Also, it is presented the dependence between those forces and the centre of mass position, the movement velocity, the curvature radius of trajectory, and the influence of suspensions characteristics and of angle between the velocity of mass centre and the cars longitudinal axe over the vertical forces.

It is analyzing those results and for external side of body an increasing loading is observed, the most loading wheel is the front wheel from external curvature. In the same time, it is observed that the rear wheel internal of curvature has an unloading. In paper, it is also determinate the critical velocity, wherefore the vertical forces at the rear wheel internal of curvature is become null and the danger of slide-slip is emergent.

2. THEORETICAL CONSIDERATIONS

The vehicle is related to a proper reference coordinate system chooses so that:

- the O origin of the coordinate system belongs to the vehicles centre mass,
- the Ox axe belongs the horizontal plane and it is parallel with the road,
- the Oy axe to be perpendicular to the longitudinal plane of the vehicle,
- the Oz axe to be included in longitudinal plane of the vehicle and it is perpendicular to road

Beside that, by reason of symmetry, it is accepted this axes are principal axes of inertia.

The moving equations of vehicle are induced by principium of mechanics. In order to determinate the interactions forces between wheels and road the derivative impulse axiom and the derivative kinetic moment are used:

$$m \cdot \vec{a}_c = \vec{F} + \sum_{i=1}^{4} \vec{F}_i \tag{1}$$

$$\overset{\Rightarrow}{J}_c \cdot \vec{\varepsilon} + \vec{\omega} \times (\overset{\Rightarrow}{J}_c \, \vec{\omega}) = \vec{M}_C + \sum_{i=1}^{4} \vec{M}_{Ci} \tag{2}$$

with:

- mass;
- $\vec{a}_c$ is the acceleration of vehicles m is the vehicle mass centre;
- $\vec{F}$ is the resultant of external forces, different by the interactions forces between wheels and the road;
- $\vec{F}_i$, i=1...4 are the interactions forces between wheels and the road;
- $\overset{\Rightarrow}{J}_c$ is the inertial tensor of vehicle, related to the proper chosen frame;
- $\vec{\omega}$ is the angular velocity of the vehicle;
- $\vec{\varepsilon}$ is the angular acceleration of the vehicle;
- $\vec{M}_C$ is the resultant moment in mass centre, due to the external forces and external moments, different by those that appear at the interaction between wheels and road;
- $\vec{M}_{Ci}$, i=1...4 are the resultant moment in mass centre due to the interaction forces between wheels and the road;

It is obtained an undetermined system of equations by projecting the (1) and (2) equations on the axes of the proper frame. The obtained system of equations is undetermined because the number of unknown parameters is bigger then the number of equations. To eliminate this inconvenient is necessary to taken into account the rolling and the pitching oscillations and the constructive characteristics of suspension.

In this way is considered that the vertical reactions depend on the suspensions deformations, also on the velocity of deformations of suspensions elements.

It will be considered the case of passing from linear movement to circular movement.

Also, it will be considered that the suspended mass, at the initial moment, does effectuate neither rolling oscillations nor pitching oscillations. This oscillation will appear as a result of the circular movement for the mass centre of vehicle. In this way, the pitching of suspended mass unto the horizontal plane will produced deformations of the suspension, and those deformations will act on the vertical reaction forces.

In those conditions, it is obtained the next system of equations regarding a vehicle during a circular movement with constant velocity and constant radius:

$$F_{1z}=\frac{m\cdot g\cdot b}{2(a+b)}-\frac{m\cdot h\cdot b}{e(a+b)}\cdot\frac{v^2\cos\gamma}{R}\left[1-e^{-\alpha_1 t}\left(\frac{\alpha_1}{\beta_1}\sin\left(\beta_1 t\right)+\cos\left(\beta_1 t\right)\right)\right]+$$

$$+\frac{m\cdot h}{2(a+b)}\cdot\frac{v^2\sin\gamma}{R}\left[1-e^{-\alpha_2 t}\left(\frac{\alpha_2}{\beta_2}\sin\left(\beta_2 t\right)+\cos\left(\beta_2 t\right)\right)\right]-\xi\cdot\frac{m\cdot h\cdot b}{e(a+b)}\cdot\frac{v^2\cos\gamma}{R}\cdot\frac{\alpha_1^2+\beta_1^2}{\beta_1}\cdot\tag{3}$$

$$\cdot e^{-\alpha_1 t}\cdot\sin\left(\beta_1 t\right)+\xi\cdot\frac{m\cdot h}{2(a+b)}\cdot\frac{v^2\sin\gamma}{R}\cdot\frac{\alpha_2^2+\beta_2^2}{\beta_2}\cdot e^{-\alpha_2 t}\cdot\sin\left(\beta_2 t\right)$$

$$F_{2z}=\frac{m\cdot g\cdot b}{2(a+b)}+\frac{m\cdot h\cdot b}{e(a+b)}\cdot\frac{v^2\cos\gamma}{R}\left[1-e^{-\alpha_1 t}\left(\frac{\alpha_1}{\beta_1}\sin\left(\beta_1 t\right)+\cos\left(\beta_1 t\right)\right)\right]+$$

$$+\frac{m\cdot h}{2(a+b)}\cdot\frac{v^2\sin\gamma}{R}\left[1-e^{-\alpha_2 t}\left(\frac{\alpha_2}{\beta_2}\sin\left(\beta_2 t\right)+\cos\left(\beta_2 t\right)\right)\right]+\xi\cdot\frac{m\cdot h\cdot b}{e(a+b)}\cdot\frac{v^2\cos\gamma}{R}\cdot\frac{\alpha_1^2+\beta_1^2}{\beta_1}\cdot\tag{4}$$

$$\cdot e^{-\alpha_1 t}\cdot\sin\left(\beta_1 t\right)+\xi\cdot\frac{m\cdot h}{2(a+b)}\cdot\frac{v^2\sin\gamma}{R}\cdot\frac{\alpha_2^2+\beta_2^2}{\beta_2}\cdot e^{-\alpha_2 t}\cdot\sin\left(\beta_2 t\right)$$

$$F_{3z}=\frac{m\cdot g\cdot a}{2(a+b)}-\frac{m\cdot h\cdot a}{e(a+b)}\cdot\frac{v^2\cos\gamma}{R}\left[1-e^{-\alpha_1 t}\left(\frac{\alpha_1}{\beta_1}\sin\left(\beta_1 t\right)+\cos\left(\beta_1 t\right)\right)\right]-$$

$$-\frac{m\cdot h}{2(a+b)}\cdot\frac{v^2\sin\gamma}{R}\left[1-e^{-\alpha_2 t}\left(\frac{\alpha_2}{\beta_2}\sin\left(\beta_2 t\right)+\cos\left(\beta_2 t\right)\right)\right]-\xi\cdot\frac{m\cdot h\cdot a}{e(a+b)}\cdot\frac{v^2\cos\gamma}{R}\cdot\frac{\alpha_1^2+\beta_1^2}{\beta_1}\cdot\tag{5}$$

$$\cdot e^{-\alpha_1 t}\cdot\sin\left(\beta_1 t\right)-\xi\cdot\frac{m\cdot h}{2(a+b)}\cdot\frac{v^2\sin\gamma}{R}\cdot\frac{\alpha_2^2+\beta_2^2}{\beta_2}\cdot e^{-\alpha_2 t}\cdot\sin\left(\beta_2 t\right)$$

$$F_{4z}=\frac{m\cdot g\cdot a}{2(a+b)}+\frac{m\cdot h\cdot a}{e(a+b)}\cdot\frac{v^2\cos\gamma}{R}\left[1-e^{-\alpha_1 t}\left(\frac{\alpha_1}{\beta_1}\sin\left(\beta_1 t\right)+\cos\left(\beta_1 t\right)\right)\right]-$$

$$-\frac{m\cdot h}{2(a+b)}\cdot\frac{v^2\sin\gamma}{R}\left[1-e^{-\alpha_2 t}\left(\frac{\alpha_2}{\beta_2}\sin\left(\beta_2 t\right)+\cos\left(\beta_2 t\right)\right)\right]+\xi\cdot\frac{m\cdot h\cdot b}{e(a+b)}\cdot\frac{v^2\cos\gamma}{R}\cdot\frac{\alpha_1^2+\beta_1^2}{\beta_1}\cdot\tag{6}$$

$$\cdot e^{-\alpha_1 t}\cdot\sin\left(\beta_1 t\right)-\xi\cdot\frac{m\cdot h}{2(a+b)}\cdot\frac{v^2\sin\gamma}{R}\cdot\frac{\alpha_2^2+\beta_2^2}{\beta_2}\cdot e^{-\alpha_2 t}\cdot\sin\left(\beta_2 t\right)$$

With the next notations:

- g is the acceleration of gravity;
- a is the distances between the mass centre and the front axle;
- b is the distances between the mass centre and the rear axle;
- e is the vehicle gauge;
- h – the centre mass height;
- R – the curvatures radius of the centre masses trajectory;
- v – the velocity of the vehicle;
- γ - is the angle between the vehicles velocity and the vehicle longitudinal axe;
- α_1, β_1 - the real part, respectively the imaginary part of the characteristic equations roots of the rolling oscillations;
- α_2, β_2 - the real part, respectively the imaginary part of the characteristic equations roots of the pitching oscillations;
- ξ - coefficient who taking into account the springiness constants and the damping coefficients of the vehicles suspension;

The anterior relations permit us to determinate the vertical forces and it is observed that the vertical forces of the rear wheel internal of curvature is decreasing.

This vertical force may become null and the due velocity is given by:

$$v = R \cdot \sqrt{\frac{eg}{h\left(2\sqrt{R^2 - b^2} + e\right)}}. \tag{7}$$

3. NUMERICAL APPLICATION

J_x =250 kgm^2; J_y =900 kgm^2; m=1300 kg;
a= 1,15 m; b= 1,35 m; e=1,4 m;
$k_1 = k_2$ =12000 N/m; $k_3 = k_4$ =14000 N/m;
$c_1 = c_2$ =1200 Ns/m; $c_3 = c_4$ =1200 Ns/m; $\xi = 0{,}1$;

The radius of curvature of circular movement is R=30 m, and the velocity of vehicle is 10 m/s, 12 m/s and the third case is for the 15 m/s velocity.

In the figure 1 it is presented the vertical force variation upon the time, for the front wheel internal to curvature.

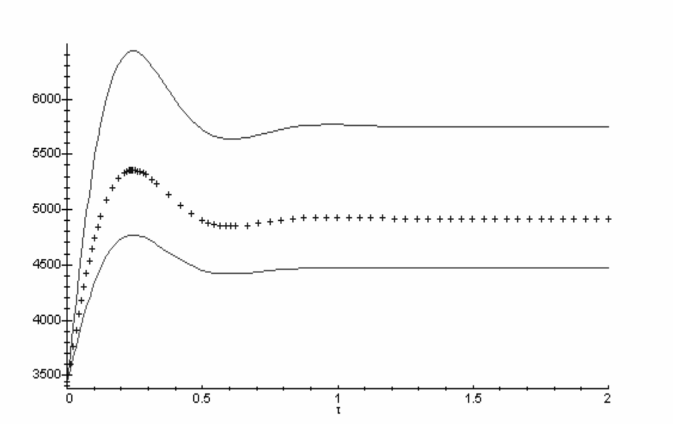

Figure 1

In the figure 2 it is presented the vertical force variation upon the time, for the front wheel external to curvature.

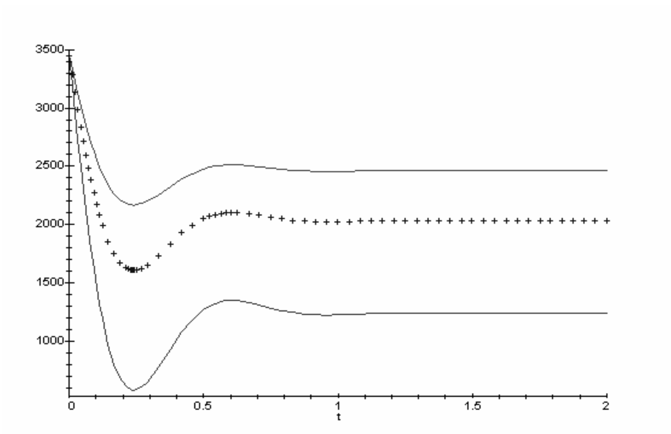

Figure 2

In the figure 3 it is presented the vertical force variation upon the time, for the rear wheel internal to curvature.

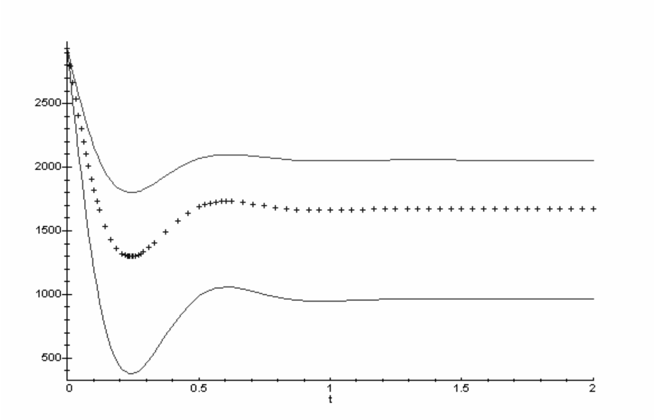

Figure 3

In the figure 4 it is presented the vertical force variation upon the time, for the rear wheel external to curvature.

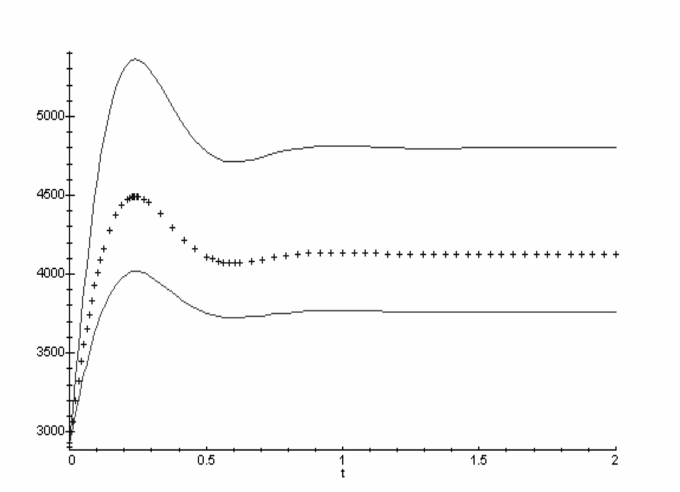

Figure 4

The next conclusions are deduced by analysing the anterior figures:

- During the circular movement, the wheels from external curvature are additional loading; the maximum value of loading is obtained for the front wheel.
- The wheels from internal curvature are unloading; the maximum value is obtained for the rear wheel. For velocity bigger then the critical velocity given by (7) relation the vertical force from the rear wheel internal to curvature becomes null, and the danger of side-slip appears.
- It is possible to appear the losing of contact between wheel and the road for a short of time and for a velocity smaller then critical velocity, due to oscillations of the vehicle.

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