# A CONTRIBUTION TO SHOCK ABSORBER MODELING BY USING "BLACK BOX" METHOD 

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#### Abstract

Shock absorbers are fundamental part of the vehicle suspension. Suspensions are needed to guarantee vehicle handling and passenger comfort [15-18,20,21,25,26]. For good handling and braking performance of the vehicle, the tire-road contact forces need to be as stable as possible. Each wheel should always remain in contact with the ground [21]. Comfort means that vibrations, induced by road profile during riding, are of a minimal nuisance to the passengers. When designing a new vehicle, a lot of development effort is focused on the optimal choice of the suspension parameters, stiffness and damping. This paper presents some results of an experimental study conducted on three shock absorbers that are in current production at Magneti-Marelli. Experimental tests were performed in conditions of random excitation. Based on the characteristic diagrams that correlate Forces with the kinematic values (Displacement, Velocity, Acceleration), by means of the "black box" method a mathematical model of the shock absorber response has been identified.


Key words: Vehicle, Shock absorber, Modelling.

## INTRODUCTION

Automotive shock absorbers are part of the vehicle suspension. Suspension are needed to guarantee vehicle handling and passenger comfort [15-18, 20, 21, 25, 26]. For a good handling and braking performance, the tire-road contact forces need to be as stable as possible. Each wheel should always remain in contact with the ground [21]. Comfort means that vibrations, induced by road profiles during riding, are of a minimal nuisance to the passengers.

When designing a new vehicle, a lot of development effort is focused on the optimal choice of the suspension parameters, stiffness and damping. A first tuning can be achieved by implementing a full car model and simulating typical road profiles $[15-18,20,21,25,26]$. The response from the simulations can give an idea about the quality of the suspension. However, the significance of the results strongly depends on the accuracy of the model.

The shock absorber is one of the most complex parts to model of the vehicle suspension $[1,13,14,19$, $22,24,27]$. In general the shock absorber behaves in a non-linear and time-variant way. Dampers are typically characterized by a simpler force-velocity diagram, also referred to as the damper characteristic diagram. Some information can also be extracted by plotting forces as a function of displacements resulting in a diagram, that in the automotive industry world is known as work diagram or resistance curve or control diagram [13].

The dependency of the shock absorber characteristics on time is due to the progressive rise of the oil temperature during the vehicle operation, which, in turn, is due to the conversion (dissipation) of kinetic energy into heat by viscous losses. Oil viscosity is a determining factor of the shock absorber characteristics and is strongly influenced by the temperature.

This paper presents results of experimental study of three shock absorbers that are in current production at Magneti-Marelli. The experimental tests were performed in conditions of random
excitation. Based on the characteristic diagrams that correlates Forces with the kinematic values (Displacement, Velocity, Acceleration) and by means of the "black box" method a mathematical model of the shock absorber response has been identified. The procedure and the obtained results will be discussed in detail in following text.

## EXPERIMENT

Measurements were conducted in Magneti Marelli plant with the MTS testing machine, that is able to provide random excitation signals. During the tests the force and displacement values are acquired from the MTS machine transducers and stored inside a PC; moreover also the acceleration of the rod and of the absorber body are measured with two acceleration sensors Bruel\&Kjajer. Relative acceleration is calculated as their difference.

Relative velocity can be calculated in two ways. Both of these ways are based on the substitution of the differential with the finite difference ratio. The first way consists in the differentiation of the relative displacement in respect to time, the second way consists in the integration of the measured relative acceleration with respect of time.
In both cases it is clear that an estimation of the true value of the velocity is obtained. As the random excitation is characterised by a spectrum that has the highest frequency component of the same order of magnitude of the sampling rate, the integration procedure and the derivation one lead to values of the velocity that are close each to the other but not identical.


Figure 1. The relation between two procedures of velocity calculation

Figure 1 shows a diagram to illustrate the relation between the velocity calculated by derivation of displacements and the velocity calculated by integration of the measured relative accelerations. From Figure 1 it is clear that there is a good correlation between two procedures, but none leads to the correct evaluation of the velocity.

Relative acceleration can be calculated both from two accelerations registered during the experimental tests (piston and body) and by means of velocity derivation in respect to time. The relation between them is shown in figure 2.

From Figure 2 it is clear that there is a good correlation between the two procedures. After some analyses and discussion, bearing in mind that, due to the previously discussed limitations, none of the procedures can lead to the true values of the kinematic quantities but it is highly desirable to unify the
methods for calculation of the velocities and accelerations, we decided to calculate relative acceleration by velocity derivation in respect to time.


Figure 2. The relation between two calculation procedures for relative acceleration

Three types of shock absorbers of Magneti Marelli current production were tested in this experiment (we will refer to them as A, B and C), and they were without rubber joints.

For the shock absorber excitation, random signals with 5 and 10 mm RMS were used. The excitation frequencies were mainly in interval from 0.1 to 20 Hz .

During experiment we used sampling time step of 0.00195 s , sample size results to be of 30722 points, that ensures the suitability of the acquired results in the interval $1.6929 \operatorname{Exp}(-4)$ to 250 Hz (the Nyquist frequency), that is acceptable when we have in mind the aim of this study [2-4].

For illustration, Figures 3 and 4 show respectively the history of displacement and force in time, for shock absorber "A" and excitation magnitude of 5 mm RMS.


Figure 3. Displacement time history

The frequency content of the displacement history (see Figure 3) is given in Figure 5 for example. We can see that the excitations are mainly in the interval 0.1 to 20 Hz with random amplitudes.


Figure 4. Force time history


Figure 5. The spectrum of displacements for shock absorber " $A$ " and 5 mm RMS magnitude of excitations
For illustration in figures 6 to 8 the forces as functions of displacement, velocity and acceleration, for shock absorber "A" and excitation with RMS 5 mm , are given.


Figure 6. Force as function of displacements


Figure 7. Force as function of velocity


Figure 8. Force as function of acceleration

## SHOCK ABSORBER MODELING

In our research, we knew only the functional dependence of the forces in respect to displacements, velocities and accelerations. Analysis of the diagrams force-displacement, force-velocity, and forceacceleration have shown that displacement, velocitiy and acceleration affect the force in shock absorber, and that should be in mind during its modeling.

Because we will not enter into the interior of shock absorbers, the task is defined as "black box" problem [6]. Namely, we knew the inputs (displacements, velocities and accelerations) and output variable (forces restituted by the shock absorbers). The goal is to define the best mathematical model of the shock absorber. The problem was divided into two parts:

1. defining the structure of the model (influential parameters: displacement, velocity, acceleration) and 2. identify the parameters of the model.

The first task is solved by using a number of possible models given by expressions (1-11).

$$
\begin{align*}
& F=x[1]+x[2] \cdot v+x[3] \cdot v^{2}  \tag{1}\\
& F=x[1]+x[2] \cdot v+x[3] \cdot v^{2}+x[4] \cdot v^{3}  \tag{2}\\
& F=x[1]+x[2] \cdot v+x[3,4] \cdot v^{2} \operatorname{sign}(v)  \tag{3}\\
& F=\left[x[1]+x[2] \cdot v+x[3,4] \cdot v^{2} \operatorname{sign}(v)\right] \cdot(x[5]+x[6] \cdot d)  \tag{4}\\
& F=\left[x[1]+x[2] \cdot v+x[3,4] \cdot v^{2} \operatorname{sign}(v)\right] \cdot(x[5]+x[6] \cdot a)  \tag{5}\\
& F=\left[x[1]+x[2] \cdot v+x[3,4] \cdot v^{2} \operatorname{sign}(v)\right] \cdot(x[5]+x[6] \cdot d) \cdot(x[7]+x[8] \cdot a)  \tag{6}\\
& \left.F=\left[x[1]+x[2] \cdot v+x[3,4] \cdot v^{2} \operatorname{sign}(v)\right] \cdot\left[x[5]+x[6] \cdot d+x[7] \cdot d^{2} \operatorname{sign}(d)\right)\right]  \tag{7}\\
& \left.F=\left[x[1]+x[2] \cdot v+x[3,4] \cdot v^{2} \operatorname{sign}(v)\right] \cdot\left[x[5]+x[6] \cdot d+x[7] \cdot a^{2} \operatorname{sign}(a)\right)\right]  \tag{8}\\
& F=\left[x[1]+x[2] \cdot v+x[3,4] \cdot v^{2} \operatorname{sign}(v)\right] \cdot\left(x[5]+x[6] \cdot e^{-|d|}\right) \quad(9)  \tag{9}\\
& \left.F=\left[x[1]+x[2] \cdot v+x[3,4] \cdot v^{2} \operatorname{sign}(v)\right] \cdot\left(x[5]+x[6] \cdot e^{-|d|}\right) \cdot\left(x[7]+x[8] \cdot e^{-|a|}\right]\right)  \tag{10}\\
& \left.F=\left[x[1]+x[2,3] \cdot v+x[4,5] \cdot v^{2} \operatorname{sign}(v)\right] \cdot\left(x[6]+x[7] \cdot e^{-|d|}\right) \cdot\left(x[7]+x[8] \cdot e^{-|a|}\right]\right)  \tag{11}\\
& F=\left[x[1]+x[2,3] \cdot v+x[4] \cdot v^{2} \operatorname{sign}(v)\right] \cdot\left[x[5]+x[6] \cdot \operatorname{th}\left(\frac{d}{3 \cdot \sigma_{d}}\right)\right] \cdot\left[x[7]+x[8] \cdot \operatorname{th}\left(\frac{a}{3 \cdot \sigma_{a}}\right)\right] \tag{12}
\end{align*}
$$

where:
$x[i], i=1,9-p a r a m e t e r s ~ o f ~ m o d e l ~ t h a t ~ s h o u l d ~ b e ~ i d e n t i f i e d, ~$
d,v,a - relative displacement, velocity and acceleration, respectively, and
$\sigma_{d}, \sigma_{a}$ - variance of displacement and acceleration, respectively.

The second task is solved by using the optimisation method, which will be the described below.
As it is well known, the method of 'stochastic parametric optimisation' is based on the methods of non-linear programming. In the optimisation process, where constraints for design parameters are present, the problem is solved by the introduction of 'external' or 'internal' penalty functions. In the specific case, for the identification of the parameters of the shock absorber, a method of 'stochastic parametric optimisation' was applied [5-7,10-12], based on the Hooke Jeeves method and "external " penalty functions [5]. The block scheme of the procedure is given in Fig. 9 [6-7, 10-12]. Since this optimisation method has been described in details in [5-7, 10-12], its description will not be repeated here. The optimisation method was programmed in Pascal language.

During the identification of unknown parameters of proposed models, we had set goal that the models should be acceptable for both levels of excitation functions. As in the experiments we had two levels of excitation, the objective function is defined by expression:
$\phi=\alpha_{1} \cdot \sum_{1}^{N}\left(F_{\bmod e l}[i]-F_{\operatorname{exper} 1}[i]\right)^{2}+\alpha_{2} \cdot \sum_{1}^{N}\left(F_{\bmod e l}[i]-F_{\operatorname{exper} 2}[i]\right)^{2}$
where:

- $\alpha_{1}, \alpha_{2}$ are weight coefficients that describe the influence rank of the corresponding excitation on the objective function. In this case we used $\alpha_{1}, \alpha_{2}=1$.
- $\quad F_{\text {model }}, F_{\text {exper } 1}, F_{\text {exper } 2}$ are the values of the forces given by model (1-12), and forces given by experimental measurements with excitation level 5 or 10 mm RMS, respectively,
- N - is the sample size (30722 points).


Figure 9 - Block diagram of the developed procedure for calculation of the model parameters
Since there are parameter constraints in practice, the optimisation was carried out by means of the introduction of defined domain of parameters:

$$
x[s]_{\min }=-5000, x[s]_{\max }=5100 *
$$

(* the measure unit depends on the parameters).
As stated in [5-7,10-12], a global (absolute) minimum of the objective function is defined in such a manner that optimal parameters lead to minimum value of the objective function. Unfortunately, there are no generally accepted procedures for definition of the global minimum of the objective function. Thus, procedures based on optimisation with different initial values of the parameters to be optimised are used in practice. In practice, the global minimum is searched by beginning the optimisation process with several different initial values of parameters to be optimised [6,7,10-12]. In this case, the optimisation process has begun with three different initial values of those parameters. Preliminary
analysis showed that only the initial values of model parameters chosen in the central part of the considered interval allow for unimpeded iterative process, and we used only these initial values.

The identification was performed on a Pentium-4 computer (Intel $1.8 \mathrm{GHz}, 1 \mathrm{~Gb}$ RAM), and the iteration process was interrupted when the difference between two adjacent values of the objective function reached $\boldsymbol{1} \boldsymbol{e}^{-09}$. Duration time of the identification per combination was approximately less than one hour. Minimum of objective functions for all the models are given in Table 1.

## DISCUSSION

Analysis of data from Table 1 shows that the structure of the model (1-12) affects the minimum value of the objective function. This is understandable, because we define the problem as a "black box" where we adopted that the better model is characterised by the smaller value of the objective function. Models (6) and (12) have values of the objective function lower than the other models.

Table 1. Minimal values of the objective functions of the models

| Model No | Shock absorber | Minimal value of the objective function |
| :---: | :---: | :---: |
| 1 | A | $4.933639923395167 \mathrm{E}+004$ |
| 1 | B | $2.731528581895664 \mathrm{E}+004$ |
| 1 | C | $1.469308043138020 \mathrm{E}+004$ |
| 2 | A | $3.076775807535464 \mathrm{E}+004$ |
| 2 | B | $1.802670955044523 \mathrm{E}+004$ |
| 2 | C | $7.656719916029218 \mathrm{E}+003$ |
| 3 | A | $2.396113131415655 \mathrm{E}+004$ |
| 3 | B | $1.500545381768893 \mathrm{E}+004$ |
| 3 | C | $5.382619164907592 \mathrm{E}+003$ |
| 4 | A | $2.394590457506216 \mathrm{E}+004$ |
| 4 | B | $1.486550659439362 \mathrm{E}+004$ |
| 4 | C | $5.319625071550819 \mathrm{E}+003$ |
| 5 | A | $2.397139551180624 \mathrm{E}+004$ |
| 5 | B | $1.487545181083031 \mathrm{E}+004$ |
| 5 | C | $5.322489965605322 \mathrm{E}+003$ |
| 6 | A | $2.395713215491265 \mathrm{E}+004$ |
| 6 | B | $2.989236747878551 \mathrm{E}+004$ |
| 6 | C | $1.822128042424834 \mathrm{E}+004$ |
| 7 | A | $6.663209154654681 \mathrm{E}+004$ |
| 7 | B | $2.653453878109062 \mathrm{E}+005$ |
| 7 | C | $1.775635220529670 \mathrm{E}+004$ |
| 8 | A | $6.891364717297212 \mathrm{E}+004$ |
| 8 | B | $2.985659561455255 \mathrm{E}+004$ |
| 8 | C | $1.807358450240563 \mathrm{E}+004$ |
| 9 | A | $2.392263130953798 \mathrm{E}+004$ |
| 9 | B | $1.504333619890715 \mathrm{E}+004$ |
| 9 | C | $5.367231209991907 \mathrm{E}+003$ |
| 10 | A | $2.390640716472426 \mathrm{E}+004$ |
| 10 | B | $1.495530969977917 \mathrm{E}+004$ |
| 10 | C | $5.385663672521046 \mathrm{E}+003$ |
| 11 | A | $1.581119622794756 \mathrm{E}+004$ |
| 11 | B | $1.737267729270784 \mathrm{E}+004$ |
| 11 | C | $8.656824628392669 \mathrm{E}+003$ |


| 12 | A | $2.392476531437416 \mathrm{E}+004$ |
| :--- | :--- | :--- |
| 12 | B | $1.471878970195807 \mathrm{E}+004$ |
| 12 | C | $5.252016484044151 \mathrm{E}+003$ |

We will discuss these results in following sections.
From Table 1 it is clear that design of the shock absorbers (A, B, C) affects the parameters of the model, and we should analyse the objective function for all the shock absorbers. Bearing that in mind, we consider that the minimal averaged value for the shock absorbers ( $\mathrm{A}, \mathrm{B}, \mathrm{C}$ ) has model No (12) and we concluded that this model is the most acceptable for the analysis. Based on this choice, the parameters for accepted model (No 12) are shown in Table 2.

Table 2. The identified parameters of the model

| Model No | Shock Absorber | Minimal value of objective function | Identified parameters of the model |
| :---: | :---: | :---: | :---: |
| 12 | A | $2.392476531437416 \mathrm{E}+004$ | -2.002903394046352E+002 |
|  |  |  | 9.511512333050106E+000 |
|  |  |  | -8.845943376824775E-003 |
|  |  |  | -2.327815204673694E-002 |
|  |  |  | $3.028817185624795 \mathrm{E}+000$ |
|  |  |  | -7.597706204283637E-002 |
|  |  |  | $2.323432897457028 \mathrm{E}-001$ |
|  |  |  | $7.847125998270326 \mathrm{E}-004$ |
| 12 | B | $1.471878970195807 \mathrm{E}+004$ | $-1.080773380138234 \mathrm{E}+002$ |
|  |  |  | $1.963536464604005 \mathrm{E}+000$ |
|  |  |  | -2.527412394532334E-003 |
|  |  |  | -3.879692017049664E-003 |
|  |  |  | $7.309897516504849 \mathrm{E}+000$ |
|  |  |  | -4.020452866363680E-001 |
|  |  |  | $3.319189853583900 \mathrm{E}-001$ |
|  |  |  | $2.017178682023871 \mathrm{E}-002$ |
| 12 | C | $5.252016484044151 \mathrm{E}+003$ | -1.939064710283242E+002 |
|  |  |  | $3.377991211657644 \mathrm{E}+000$ |
|  |  |  | -4.422971689244961E-003 |
|  |  |  | -7.759384022987308E-003 |
|  |  |  | $1.207000621811347 \mathrm{E}-001$ |
|  |  |  | -5.698279654720194E-003 |
|  |  |  | $9.364857087367026 \mathrm{E}+000$ |
|  |  |  | $4.703949717392524 \mathrm{E}-001$ |

For analysis purpose, some signals have been processed with softwares "Analsigdem" and "Demparcoh" [8,9].

For illustration, in figures 10-19 some results for shock absorber (A) are shown.
From the analysis of all data, which are partially shown in the figures 10-12 and 15-17, we can conclude that there is good coincidence between results obtained by experiment and model.

It should be noted that for very negative values of forces for excitation RMS 10 mm , for all shock absorbers, large differences occur in the function of speed. That can be explained by the influence of non-linearity of the system.

Finally, we analysed coherence functions that are partially shown in Figure 14 and 19. Analysis showed that there is acceptable value of the aforementioned functions, so that the model (12) can be suitably used in vehicle dynamic simulations.


Figure 10. Comparison of forces measured in experimental tests and computed with the identified model in respect to displacement for excitation RMS $5 \mathbf{~ m m}$


Figure 11. Comparison of forces measured in experimental tests and computed with the identified model in respect to velocity for excitation RMS 5 mm


Figure 12. Comparison of forces measured in experimental tests and computed with the identified model in respect to acceleration for excitation RMS 5 mm


Figure 13. Comparison of forces measured in experimental tests and computed with the identified model for excitation RMS $5 \mathbf{~ m m}$


Figure 14. Coherence function between the forces measured in experimental tests and computed with the identified model for excitation RMS $5 \mathbf{~ m m}$


Figure 15. Comparison of forces measured in experimental tests and computed with the identified model in respect to displacement for excitation RMS 10 mm


Figure 16. Comparison of forces measured in experimental tests and computed with the identified model in respect to velocity for excitation RMS 10 mm


Figure 17. Comparison of forces measured in experimental tests and computed with the identified model in respect to acceleration for excitation RMS 10 mm


Figure 18. Comparison of forces measured in experimental tests and computed with the identified model for excitation RMS 10 mm


Figure 19. Coherence function between the forces measured in experimental tests and computed with the identified model for excitation RMS 10 mm

## CONCLUSION

Bearing in mind previous analyses, we can conclude that the method "black box" and optimisation methods for identifying of parameters of the shock absorber model are acceptable. In this study the model described as (12) is acceptable for vehicles dynamic simulation.

## REFERENCES

1. Audenino, A., Belingardi, G., Garibaldi, L.: An application of the restoring force mapping method for the diagnostic f vehicular shock absorbers dynamic behaviour, 2nd Int. Machinery monitoring and diagnostic conference \& exibit, LA, 1990, pp. 560-566.
2. Bendat J.S.: Nonlinear Systems-Techniques and Applications, John Wiley and Sons, London, 1998.
3. Bendat J.S., Piersol A.G.: Random Data Analysis and Measurement Procedures, John Wiley and Sons, London,2000.
4. Bendat J.S, Piersol A.G.: Engineering Applications of Correlation and Spectral Analysis, John Wiley and Sons, London, 1980.
5. Bunday, P.: Basic Optimization Methods, Spottiswoode Ballantyne, Colchester and London, 1984.
6. Demić M.:Identification of Vibration parameters for Motor Vehicles, Veh. Syst. Dyn. Vol. 27, 1987, pp. 65-88
7. Demić M.: Optimization of Characteristics of Elasto-Damping Elements from Aspect of Oscillatory Comfort and Vehicle Handling, Int. J. of Veh. Des. Vol. 17, № 1, 1996, pp. 76-91
8. Demić M.: "ANALSIGDEM" - Software for signal analysis, www.ptt.yu/korisnici/i/m/imizm034/,2003.
9. Demić, M.:"DEMPARCOH": Software for partial coherence function calculation, www.ptt.yu/korisnici/i/m/imizm034/ ,2003.
10. Demić, M.: Optimization of Vehicles Elasto-Damping Elements Characteristics from the Aspect of Ride Comfort, Veh. Syst. Dyn. Vol. 23, 1994.
11. Demić, M.:A contribution to optimization of vehicle seats, Int. J. of Veh. Des. 5/6, 1991, pp. 618-629.
12. Demić, M.: A contribution to the optimization of the characteristics of elasto-damping elements of passenger cars, Veh. Syst. Dyn. Vol. 19,1990, pp. 3-18
13. Duym, S., Stiens, R., reybrouck, K.: Evaluation of shock absorbers models, Vehicle System Dynamics, Vol. 27, 1997, pp. 109-127.
14. Gardulski, J., Warczek, J.: Investigation of forces in frictional kinematic pairs to acces their influence on shock absorber characteristics, Transport problems, Tom 3, 2008, pp. 19-24.
15. Genta, G.: Motor Vehicle Dynamics, Verlag, 2003.
16. Ellis, J.R.: Vehicle Handling Dynamics, Mechanical Engineering Publications Limited, London, 1994.
17. Gillespie T (1992) Fundamental of Vehicle Dynamics, SAE
18. Hačaturov, A.A. et al.: Dinamika sistemi: Doroga-Šina-Avtomobilj-Voditelj, Mašinostr., Moscow (in Russian), 1976.
19. Lee, C. T., Moon, B, Y.: Simulation and experimental validation of vehicle dynamic characteristics for displacement sensitive shock absorber using fluid-flow modelling, Mechanical Systems and signal processing, Vol. 20, 2006, pp. 373-388.
20. Miliken, W., Miliken, D.: Race Car Dynamics, SAE,1995.
21. Mitschke, M.: Dynamik der Kraftfahrzeuge, Springer, 1972.
22. Ping, Y.: Experimental and mathematical evaluation of dynamic behaviour of an oil-air coupling shock absorber, Mechanical Systems and signal processing, Vol 17, 2003, pp. 13671379.
23. Ping, Y.: Testing and mathematical approach on dynamics of Cymene-Si-Oil shock absorber for structural integrity in vibration and impact protection, Int. J. Materials and structural integrity, Vol. 1, Nos 1,2,3, 2007, pp. 1-15.
24. Pracny, V., Meywerk, M., Liona, A.: Hybrid neural network model history-dependent automotive absorbers, Vehicle System Dynamics, Vol. 45, 2007, pp. 1-14.
25. Rotenberg, V.: Vehicle suspension, Mašinostr. Moscow (in Russian), 1972.
26. Simić, D.:Motor vehicle dynamics, Naučna knjiga, Begrade (in Serbian),1988.
27. Yang, P., Liao, N., Yand, J.: Design and test and modelling of a novel Si-oil absorber for protection of electronic equipment in moving vehicles, Mechanism and Machine Theory, 2007, pp. 18-32.
