THE OPTIMIZATION OF A TRANSPORT NETWORK USING KRUSKAL'S ALGORITHM

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Abstract: *This paper presents a study regarding the optimization of a transport network based of graph theory, more exactly by determining the minimum spanning tree for a connected weighted graph using Kruskal' algorithm. Based on this algorithm, a program in C was designed and utilized to optimise a transport network for Romanian Post. The objectives were the shorter delivery times and distances.*

Keywords: optimization, transport networks, graph theory, Kruskal' algorithm

INTRODUCTION

A series of economical problems regarding the transport activity can be realized using graph theory elements, thus one may grasp the in-depth analysis. The connections between the analyzed problem's components can be elaborated more easily and accurately precision while creating a corresponding mathematical model. A particular example is the tree.
The problem of calculating a minimal route which can correspond to the imposed requirements, minimal distance and minimal delivery time within the Romanian Post transport network, has arisen. The solution of this problem was to determine the minimum spanning tree.
Most of these problems require knowledge whether the connections between its components exist (graph nodes) and also of their intensity, measured by a numerical value associated with the corresponding connection between components.

ACTUAL SITUATION

For a complete study of the transport network, it has to define the whole area of coverage in that the Romanian Post carries on its activity. Thus, the Romanian Post is spread across the whole country, in 41 counties and in the capital, Bucharest.
The Romanian Post has a network that is formed out of 10 Regional Directions that attend to the whole activity of mail and couriers:

1. Bacau
2. Brasov
3. Bucharest
4. Cluj
5. Constanta
6. Craiova
7. Galati
8. Iasi
9. Ploiesti
10. Timisoara

and five specialized directions: Expedition House, Rapid Post Direction, Financial Services Direction, The National Stamp Museum and The Stamp Factory.
For a better distribution of the transport network, as well as for its diversification, the Regional Directions contain County Offices and Regional Transit Centres. The County Offices are split in Urban Postal Offices, Centralized Distribution Post Offices, Rural Mechanized Post Offices/Post Offices with Transit Activity and Rural Post Offices. The Urban Post Offices contain Urban Postal Pay Desks.
Figure 1 presents the actual transport mode of the Romanian Post. On the map are presented all of the Regional Directions, as well as the components of each region with the corresponding County Offices represented by the red symbols. The number attached to every County Post Office represents the number of Post Offices available within it. The blue symbols coincide with the Regional Directions.

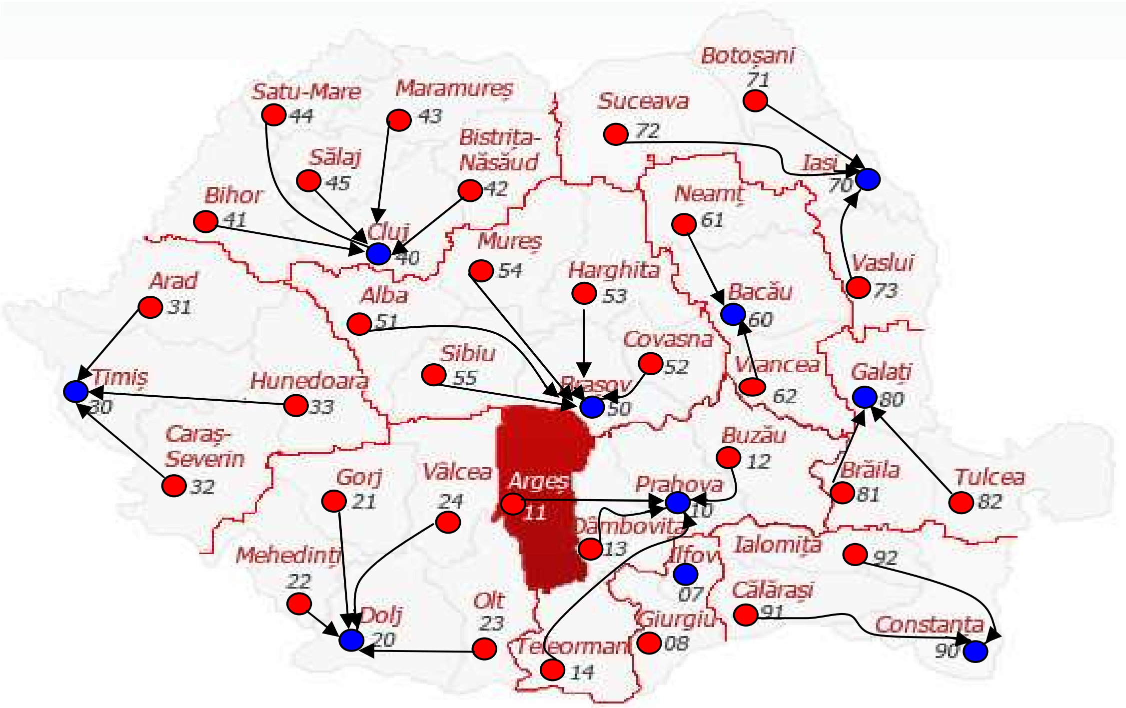

Fig. 1. The actual transport mode of the Romanian Post

First, the deliveries are collected at Post Office level. Then they are classified and sent to a County Post Office (red colour). All of the collected expeditions are directed towards and taken over by a representative Regional Post Centre for each area (blue colour), with auto vehicles, on national roads.
After all collecting is completed at Regional Directions, the objects are transported to the Bucharest Regional Direction.
The transport routes are presented in figure 2 with green lines. All of these routes at Regional Direction level end up at the common destination point, Bucharest Regional Direction.
The principal routes at Regional Direction level are mentioned in Table 1, being characterized by distance and delivery time.

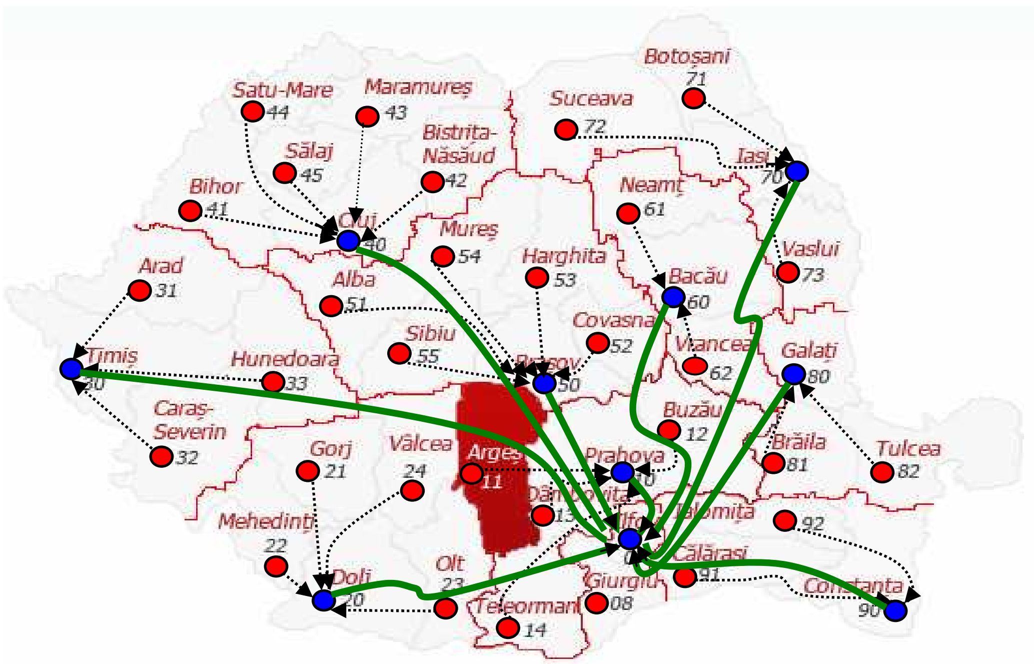

Fig. 2. The routes of the actual network used for transport

Table 1. The principal routes at Regional Direction level regarding the distance and delivery time

National level routes	Distance of transport [km]	Delivery time [h]
Bucuresti– Timisoara	535	6h15
Bucuresti – Craiova	225	2h53
Bucuresti – Cluj	441	4h57
Bucuresti – Brasov	168	1h58
Bucuresti – Prahova (Ploiesti)	57	0h42
Bucuresti – Bacau	306	3h35
Bucuresti – Iasi	409	4h52
Bucuresti – Galati	257	3h13
Bucuresti– Constanta	266	2h34

Thus the total distance for the final collecting in Bucharest Regional Direction is:

$$D_{total} = \sum_{i=1}^{n} di \quad (1)$$

where d_i represents the distance from Bucharest to each Regional Direction.
So, the total distance is:

$$D_{total} = 535 + 225 + 441 + 168 + 57 + 306 + 409 + 257 + 266 = 2664 \text{ [km]} \quad (2)$$

DESIGNING THE OPTIMIZATION PROGRAM

The problem is to determine the minimum spanning tree of a given graph. Resolving this problem means to use an algorithm that extracts the tree by adding edges one by one. The value of each edge represents the length of the road between two Regional Post Directions.
In order to find the optimal solution Kruskal's algorithm was used. G=(X,U) is a connex graph. To determine the minimum spanning tree, the algorithm chooses the edge that has the smallest value and does not form a cycle with the other edges already added. If the edge values are distinct, the solution is unique. Here are the steps of the algorithm:

1. Ordering the edges by ascending value
2. Of all the edges, the one with minimum value is chosen
3. If the chosen edge forms a cycle when added, the edge is eliminated and the algorithm goes back to step 2.
4. Step 2 is processed until (n-1) edges are added.

The minimum value of the tree is obtained when the edge values are added.
A program in C was written to use this algorithm:

```
#include <stdio.h>

typedef struct muchie {
        int x,y,c;
} muchie;

int v[100];
muchie muc[100];
char *num[255];
int n,m;

int citire() {
     int i,j,aux,m=0;
     printf("Introduceti numarul de localitati: ");
     scanf("%d",&n);
     for (i=0;i<n;i++)
         num[i]=(char*)malloc(255*sizeof(char));
     for (i=1;i<=n;i++) {
```

```
        printf("Introduceti numele localitatii nr. %d:",i);
        scanf("%s",num[i-1]);
    }
    for (i=0;i<n-1;i++)
        for (j=i+1;j<n;j++) {
            printf("Introduceti costul intre %s si %s (-1 daca nu exista legatura intre ele):",num[i],num[j]);
            scanf("%d",&aux);
            if (aux!=-1) {
               muc[m].c=aux;
               muc[m].x=i;
               muc[m].y=j;
               m++;
            }
        }
    return m;
}

int isGata() {
   int i;
   for (i=0;i<n-1;i++)
      if (v[i]!=v[i+1])
         return 0;
   return 1;
}

int Kruskal() {
    int i,j=0,nr=0,s=0;
    while (isGata()==0) {
          while (j<m && v[muc[j].x]==v[muc[j].y])
                j++;
          printf("%s - %s\n",num[muc[j].x],num[muc[j].y]);
          s=s+muc[j].c;
          nr++;
          if (nr==n-1)
             break;
          for (i=0;i<n;i++)
              if (v[i]==v[muc[j].y])
                 v[i]=v[muc[j].x];
    }
    return s;
}

int main() {

    int i,j;
    muchie aux;
    m=citire();
    for (i=0;i<n;i++)
        v[i]=i;
    for (i=0;i<m-1;i++)
        for (j=i+1;j<m;j++)
             if (muc[i].c>muc[j].c) {
                aux=muc[i];
                muc[i]=muc[j];
                muc[j]=aux;
             }
    printf("Rezultatul este: %d",Kruskal());
    getch();
    return 0;
}
```

FINDING THE MINIMUM SPANNING TREE FOR DISTANCE

There are 10 nodes representing 10 Regional Post Directions. The edges will be the values of the distances between them. The values are indicated in Table 2.

Table 2. Distances between Regional Post Directions [km]

	Bacau	Brasov	Bucuresti	Cluj	Constanta	Craiova	Galati	Iasi	Ploiesti	Timisoara
Bacau	0	177	284	385	390	432	185	134	240	619
Brasov	177	0	168	274	380	255	294	311	108	423
Bucuresti	284	168	0	440	266	234	244	393	60	562
Cluj	385	274	440	0	706	406	568	427	382	334
Constanta	390	380	266	706	0	500	205	430	272	828
Craiova	432	255	234	406	500	0	478	566	243	328
Galati	185	294	244	568	205	478	0	252	210	717
Iasi	134	311	393	427	430	566	252	0	352	734
Ploiesti	240	108	60	382	272	243	210	352	0	531
Timisoara	619	423	562	334	828	328	717	734	531	0

For a better simplification, each Regional Direction will have an associated node x_i as is indicated in Table 3.

Table 3. The correspondence of the associated nodes

Regional Directions	Associated node
Bacau	x_1
Brasov	x_2
Bucuresti	x_3
Cluj	x_4
Constanta	x_5
Craiova	x_6
Galati	x_7
Iasi	x_8
Ploiesti	x_9
Timisoara	x_{10}

Figure 3 presents the graph with the retained edges. The minimum value of this spanning tree is 1705 and it is unique.

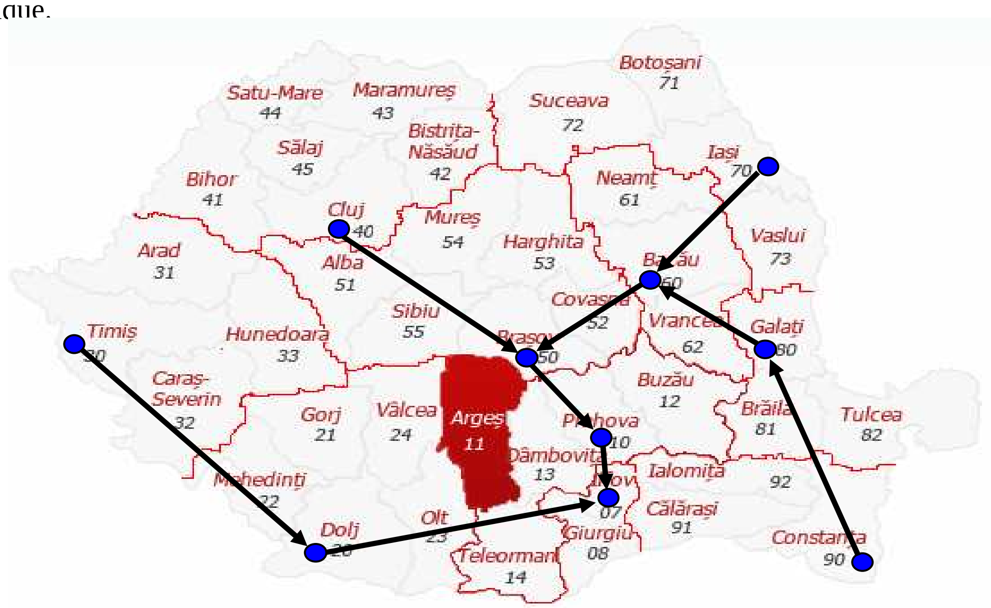

Fig. 3. The minimum distance spanning tree

After this analysis we can observe the difference between the actual transport network system and the one proposed for optimization:

- actual situation of the transport network: D_{total}=2664km
- optimized studied situation of the transport network: D_{total}=1705km.

In this case the transport distance can be reduced by 959 km which means 36% of the actual transport network.

FINDING THE MINIMUM SPANNING TREE FOR TIME

The graph will be constructed the same way as before, only the value of each edge will be delivery time between two nodes. The talked-about values are attached in Table 4.

Table 4. The time needed for moving between the Regional Post Directions [min]

	Bacau	Brasov	Bucuresti	Cluj	Constanta	Craiova	Galati	Iasi	Ploiesti	Timisoara
Bacau	0	150	215	321	299	329	133	89	183	457
Brasov	159	0	118	193	280	170	210	248	76	298
Bucuresti	215	118	0	297	154	173	193	292	42	435
Cluj	321	193	297	0	473	299	253	336	269	231
Constanta	299	280	154	473	0	327	190	372	196	529
Craiova	329	170	173	299	327	0	322	418	162	247
Galati	133	210	193	253	190	322	0	182	160	508
Iasi	89	248	292	336	372	418	182	0	260	546
Ploiesti	183	76	42	269	196	162	160	260	0	374
Timisoara	457	298	435	231	529	247	508	546	374	0

Figure 4 presents the resulted layout for which the spanning tree is minimum in terms of delivery time.

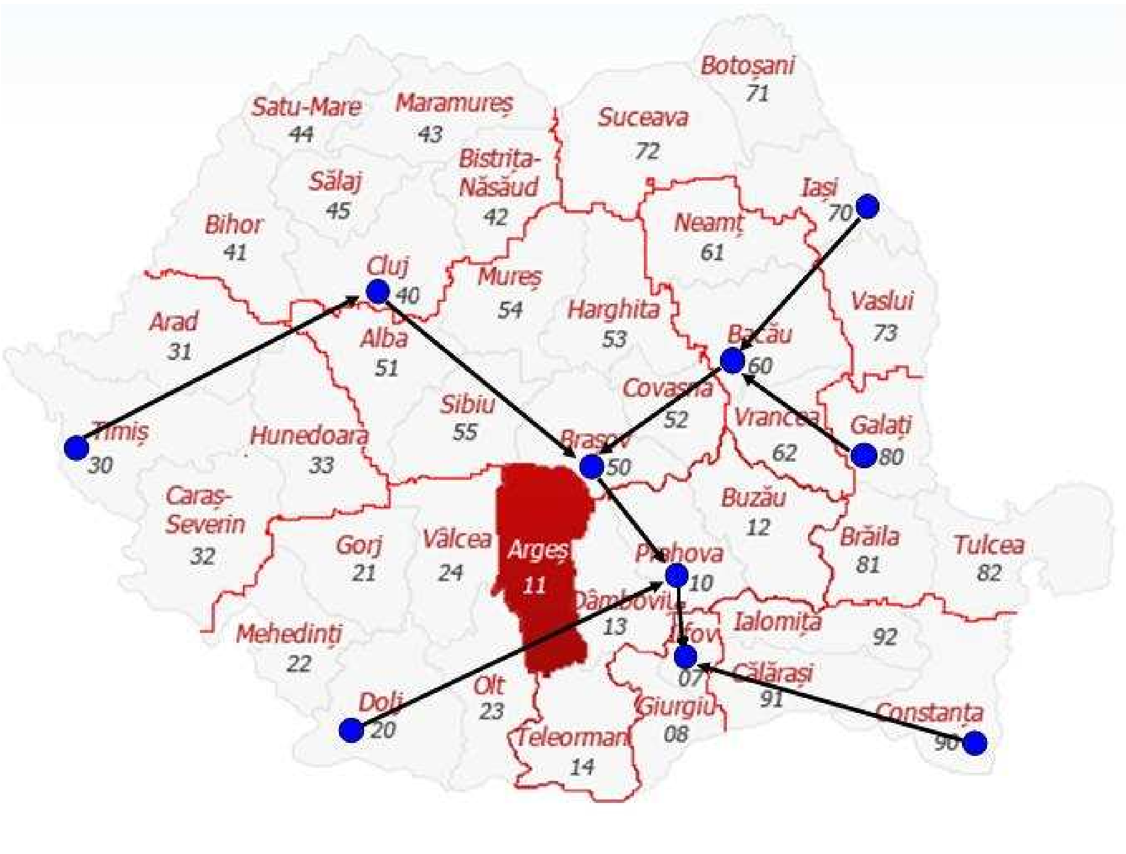

Fig. 4. The minimum time spanning tree

CONCLUSIONS

Transport activity, as an essential activity in distributing goods, has a large range of approaches, in the sense of optimizing this process. The minimization of transport distances, costs and transport duration are problems that logisticians are confronting with while trying to realize an optimal distribution of products to end-users.

The increase of efficiency in delivering goods may be realised by the dynamic optimization of the transport routes and of the utilisation of delivery vehicles, by reducing the delivery time etc.

This paper presented a study regarding the optimization of a transport network for the Romanian Post. Based on Kruskal's algorithm, a program in C was created for a practical application in order to minimize the transport distance and the delivery time.

Based on this study a consistent reserve for optimizing transport process is revealed. Thus, in the case of minimizing the transport distance, it can be reduced by 959 km (that is 36% of the actual transport network). What is interesting is the fact that by minimizing the transport network, a new transport network is created, which can be periodically brought up-to-date, depending on the evolution of road traffic.

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